Holomorphic Flows, Cocycles, and Coboundaries

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Dedicated to Professor Frank Forelli in fond remembrance

1. Introduction

Cocycles appear in many areas of analysis (harmonic analysis, representation theory, operator theory, ergodic theory, etc.) and, indeed, they are present whenever a group or a semigroup acts as a transformation group on some space. In a sense, cocycles are generalizations of the exponential function and provide a measure of "normality" of the underlying group action. We are primarily concerned with the semigroup action provided by a holomorphic flow on a domain in the complex plane.

The properties of semigroups of holomorphic flows may be studied by replacing these semigroups by any member of a large class of isospectral operators generated from the above semigroups by certain types of cocycles called *coboundaries*. This motivation has led us to investigate when cocycles are coboundaries, and in doing so, we are led to a complete description of all holomorphic flows on $\mathbb{C}$. Our approach and techniques are quite direct and independent of operator-theoretic considerations.

The relatively recent study of holomorphic flows was initiated by Berkson and Porta [BP], who showed the strong continuity of these flows on Hardy spaces. Cowen [C1] provided an interesting application of holomorphic flows on Hardy spaces to prove, among other things, that the Cesàro operator is subnormal. Siskakis [S1; S2] extended the results of [BP] to Bergman spaces and applied weighted holomorphic flows on Hardy spaces to the study of the Cesàro operator. König [Ko] investigated weighted holomorphic flows on the unit disc and gave a characterization of the smooth cocycles on the Hardy space. Some of our results complement those found in [BP] and [Ko], but our techniques are considerably different. Related ideas also appear in [EJ; F; H; J; JY; SM; Y]. An extensive article outlining the history of translation flows and their applications to dynamical systems is given by Latushkin and Stepin [LS].

Our notation and terminology are as follows. Let G be a domain (open, connected and nonempty) in the complex plane $\mathbb{C}$, and let $H(G)$ be the set of holomorphic functions on G. We shall use Δ to denote the open unit disc in $\mathbb{C}$. A one-parameter family $\varphi(t, z)$ of nonconstant holomorphic functions from G to G that satisfy $\varphi(0, z) = z$ and $\varphi(s + t, z) = \varphi(s, \varphi(t, z))$ for all $s, t \geq 0$ and $z \in G$

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