A Sufficient Condition for $\operatorname{Proj}^1 \mathcal{X} = 0$

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1. Introduction. In [7], Palamodov established a homological theory for projective spectra of topological vector spaces. In applications of this theory, it is crucial to decide whether, for a given projective spectrum $\mathcal{X}$ of (DFS) spaces, a certain vector space $\operatorname{Proj}^1 \mathcal{X}$ is trivial. A topological characterization of $\operatorname{Proj}^1 \mathcal{X} = 0$ has been given by Retakh [8]. In practical cases, its evaluation is hard. In Vogt [9; 10], more tractable conditions were given, which were motivated from the structure theory of nuclear Fréchet spaces. There is a sufficient as well as a necessary condition, but these are probably different. In the case of sequence spaces, it is shown in [9] that the necessary condition is also sufficient. Recently, Wengenroth [11; 12] has proved the sufficiency of the necessary condition also for (DFM) spectra. His proof is based on the investigation of topological properties of the dual inductive spectrum. In the present paper, we give a direct proof of Wengenroth's result for the case of (DFS) spectra. It grew out of a third condition, the sufficiency of which was shown in Braun [1].

To present an application, let $P(D)$ denote a constant coefficient partial differential operator, let $\Omega \subset \mathbb{R}^N$ be a convex domain, and denote by $A(\Omega)$ the space of all real analytic functions on Ω and by $\Gamma^d(\Omega)$ the Gevrey class of exponent d. Recall that $\Gamma^1 = A$ and that $\Gamma^d(\Omega)$ contains test functions if $d > 1$. Hörmander [5] has characterized the surjective operators $P(D)\colon A(\Omega) \to A(\Omega)$. He used a Mittag–Leffler procedure to show the sufficiency of his condition and a Baire category argument to derive necessity. Braun, Meise, and Vogt [2; 3] used Palamodov's homological approach to extend this theorem to $\Gamma^d(\mathbb{R}^N)$, $d > 1$. To do so, they proved the equivalence of Vogt's two conditions using Fourier analysis. This failed in the case of arbitrary convex domains, which was solved in Braun [1]. The functional analysis part of this proof was distilled out of the Mittag–Leffler argument in Section 5 of Hörmander [5]. Further refinement then led to the proof presented here. A very similar proof was independently found by Frerick and Wengenroth [4] by dualizing the acyclicity argument of Wengenroth [11; 12].

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2. Definition. A projective spectrum $(X_k, \iota^k_{k+1})_k$ consists of a sequence $(X_k)_k$ of vector spaces together with linear mappings $\iota^k_{k+1}\colon X_{k+1} \to X_k$. Each of the spaces X_k is the inductive limit $X_k = \bigcup_{n=1}^\infty X_{k,n}$ of a sequence $X_{k,1} \subset X_{k,2} \subset \cdots$

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