The Unitary Orbit of Strongly Irreducible Operators in the Nest Algebra with Well-Ordered Set

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1. Introduction

Let $\mathcal{H}$ be a complex, separable, infinite-dimensional Hilbert space; $\mathcal{L}(\mathcal{H})$, $\mathcal{K}(\mathcal{H})$ denote (respectively) the algebra of all bounded linear operators acting on $\mathcal{H}$ and the ideal of all compact operators.

Let $\sigma_0(T)$ denote the isolated eigenvalues of T of finite multiplicity. If λ belongs to $\sigma_0(T)$, let $E_T\{\lambda\}$ denote the Riesz projection corresponding to the eigenspace for λ. When X is a compact subset of the plane, let $X^\wedge$ denote the polynomially convex hull of X.

An operator T is *strongly irreducible* if the only idempotent operators in $\{T\}'$ are 0 and I, where $\{T\}'$, denotes the commutant of T. Let Ω be a bounded connected open set in $\mathcal{C}$. Recall that $\mathcal{B}_n(\Omega)$, the set of Cowen–Douglas operators of index n $(1 \leq n \leq +\infty)$, is the set of those operators B on $\mathcal{H}$ satisfying

(i) $\sigma(B) \supset \Omega$;
(ii) $\operatorname{nul}(\lambda - B) = \operatorname{ind}(\lambda - B) = n, \quad (\lambda \in \Omega)$;
(iii) $\bigvee\{\ker(\lambda - B); \lambda \in \Omega\} = \mathcal{H}$.

Note that (iii) can be replaced by

(iii′) $\bigvee\{\ker(\lambda_0 - B)^k : k \geq 1\} = \mathcal{H}$ for some $\lambda_0 \in \Omega$.

A nest $\mathcal{N}$ in $\mathcal{H}$ is a linearly ordered (by inclusion) family of subspaces containing $\{0\}$ and $\mathcal{H}$. The *nest algebra* associated with $\mathcal{N}$ is the family of operators defined by

$$\mathcal{T}(\mathcal{N}) = \{T \in \mathcal{L}(\mathcal{H}) : TN \subset N \text{ for all } N \text{ in } \mathcal{N}\}.$$

In what follows, $N \in \mathcal{N}$ denotes both a subspace and the orthogonal projection onto it; $T \in (\mathrm{SI})$ means that T is a *strongly irreducible* operator on its acting space.

For each $N \in \mathcal{N}$, let

$$N_- = \bigvee\{N' \in \mathcal{N},\ N' \subsetneqq N\}.$$

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