Interpolating Sequences for
Weighted Bergman Spaces of the Ball

Miroljub Jevtić, Xavier Massaneda,
& Pascal J. Thomas

0. Introduction

Let B_α^p be the space of holomorphic functions f in the unit ball $\mathbb{B}^n$ of $\mathbb{C}^n$ such that $f \in L^p((1 - |z|^2)^{\alpha - 1/p} dm)$, where $0 < p \leq \infty$ and $\alpha \geq 0$ (weighted Bergman space). In this paper we study the interpolating sequences for various B_α^p. The limiting cases $\alpha = 0$ and $p = \infty$ are respectively the Hardy spaces H^p and $A^{-\alpha}$, the spaces of holomorphic functions with polynomial growth of order α, which have generated particular interest. Note that the class of spaces we are considering is invariant under restriction to balls of lower complex dimension, which justifies the choice of those special weights.

As far as we know, for $n > 1$ the first research on this subject was carried out by Amar [Am] for the classical Bergman spaces, which in our notation correspond to the case $\alpha = 1/p$. Amar's main result states that separated sequences (in terms of the Gleason invariant distance) can be written as finite unions of interpolating sequences for $B_{1/p}^p$.

A sufficient condition due to Berndtsson [Be] is known for the case H^∞. Also, Varopoulos [Va] showed that if $\{a_k\}_k$ is H^∞-interpolating then $\sum_k (1 - |a_k|^2)^n \delta_{a_k}$ is a Carleson measure. Later, Thomas [Th1] proved that the same necessary condition holds for H^1 and that it actually characterizes the finite unions of H^1-interpolating sequences.

On the other hand, after Seip's characterization of $A^{-\alpha}$-interpolating sequences in the unit disc ([Se1], see another proof in [BO]), Massaneda [Ma] obtained some results for the case $n > 1$. In particular, $\{a_k\}_k$ is a finite union of $A^{-\alpha}$-interpolating sequences if and only if $\sum_k (1 - |a_k|^2)^{n+1} \delta_{a_k}$ is an $(n + 1)$-Carleson measure or, equivalently, if and only if $\{a_k\}_k$ is a finite union of separated sequences.

It is worth noting that in [Se1], Seip also implicitly gives a characterization of interpolating sequences for all weighted Bergman spaces in the disk. In Section 5 we spell out the details for the reader's convenience.

Here we deal with different aspects concerning B_α^p-interpolating sequences. In Section 1 we first collect some definitions and well-known facts about weighted Bergman spaces and then introduce the natural interpolation problem, along with