The Holonomy in Open Manifolds of Nonnegative Curvature

Valery Marenich

According to the well-known result by Cheeger and Gromoll [CG], every open (i.e. complete noncompact) manifold V^n of nonnegative sectional curvature $K_\sigma \geq 0$ is diffeomorphic to the space of the normal bundle νS of some totally geodesic submanifold S, called the *soul* of V^n. In this article we consider some relations between the geometry of V^n and the holonomy of this bundle. If V^n is isometric to the direct product $V^n = S \times W$ (where W is an open manifold of nonnegative sectional curvature, diffeomorphic to the Euclidean space), then the holonomy operator is the identity; that is, for every closed curve $\omega(s) \subset S$, $0 \leq s \leq 1$, the parallel translation I_ω along this curve maps every vector of $\nu_p S$ for $p = \omega(0)$ into itself. So $I_\omega = \mathrm{id}$ for every closed curve ω on S, and we will say that νS has trivial holonomy. One of the main results of this article is that the converse is also true (see Section 1).

Theorem 1. *If νS has trivial holonomy, then V^n is isometric to the direct product: $V^n = S \times W$.*

This theorem was announced in [M1].

In Section 2 we find some conditions on the behavior of the curvature near S for a trivial holonomy that, according to Theorem 1, lead to the metric splitting. Originally these conditions (Theorems 2, 3, and 4 herein) were found with the help of some geometric construction and received rather long but straightforward proofs; see [M3]. Then a very short proof of Theorem 2 was presented to the author by G. Perelman, who suggested the possibility of finding a similar short and analytic proof for Theorem 4 also. That is done at the end of Section 2.

Theorem 2. *For every point p on S and every 2-dimensional direction σ that is normal to S at this point (i.e., $\sigma \subset \nu_p S$), if*

$$K_\sigma = 0$$

then $I_\omega = \mathrm{id}$ for every contractible curve ω on S and the universal cover $\tilde{V}^n$ of V^n is isometric to the direct product.

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