Layer Potential Operators and a Space of Boundary Data for Electromagnetism in Nonsmooth Domains

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1. Introduction

Several developments in the study of singular integrals and related harmonic analysis techniques have made it possible to apply the classical method of layer potentials to numerous boundary value problems in domains with minimal regularity assumption. Among others, important examples in the case of C^1 domains are [7] and [18], and in the case of Lipschitz domains [17] and [6]. If for a given boundary value problem the classical method of layer potentials in smooth domains provides solutions for continuous boundary data, then one has come to expect that in C^1 domains, the method should also give solutions for data in L^p for the full range $1 < p < \infty$; see for example [7] and [18]. Remarkably, this should still be achieved, as in the classical case, through Fredholm theory. On the other hand, in the case of arbitrary Lipschitz domains, Fredholm theory is not applicable in general. In some cases, however, the method of layer potentials combined with energy estimates like the ones in [9] still provides solutions in the Lipschitz situation, at least for a more restrictive range of p; see for example [17] and [6].

Along these lines, we have studied in [16] several boundary value problems for the scalar Helmholtz equation $(\Delta+k^2)u=0$ in Lipschitz domains (see also [1]). This equation arises in the study of the scattering of time-harmonic acoustic waves. The purpose of this paper is to study a boundary value problem in the vector-valued case, which naturally occurs in the scattering of electromagnetic waves. We will consider the so-called Maxwell boundary value problem for the case of a perfect conducting surface, extending known results for smooth domains to domains with less regular boundaries. In the case of smooth domains, the classical theory is described in [15] and [4]. More recent developments for the Helmholtz equation in smooth domains, as well as numerous references to related works, can be found in [10] for the scalar case and in [12] for the vector-valued case.

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