Nontriviality of the Abel–Jacobi Mapping for Varieties Covered by Rational Curves

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1. Introduction

The existence and structure of families of rational curves on projective varieties has played a key role in Mori's program to classify higher-dimensional complex varieties. One of the early results in this program showed that if X is a Fano variety (i.e., the anticanonical bundle $-K_X$ of X is ample) then X is covered by rational curves (see [6]). In this paper it will be shown that if a smooth projective variety X is covered by rational curves, then these rational curves, together with their degenerations, generate the middle-dimensional primitive cohomology of X via the "cohomological" Abel–Jacobi mapping. When X is a threefold this result will be reinterpreted to give a surjectivity result for the Abel–Jacobi mapping of Griffiths into the intermediate Jacobian of X. In particular, it follows that for Fano threefolds the images of the families of rational curves C on X with $(-K_X \cdot C) \leq 4$ generate the intermediate Jacobian of X. This result validates the general principle espoused by Clemens in [4] that the intermediate Jacobian of a threefold X which is covered by rational curves is generated by algebraic cycles on X.

The literature on this subject is vast, but to the author's knowledge the results just described have only been obtained for low-degree rational curves on special Fano varieties. For example, the result is known for the families of lines on several generic complete-intersection Fano threefolds [2; 14; 5] and for generic hypersurfaces of degree n in $\mathbb{P}^n$ [12]. It is also known for conic bundles [1] and for the families of conics on a generic quartic threefold and sextic double solid [11; 3]. In each case, the arguments used to obtain nontriviality of the Abel–Jacobi mapping use specific facts about the family of curves in question or their degenerations, and for this reason do not carry over to more general situations.

The cohomological Abel–Jacobi mapping is defined as follows. Let X be a smooth complex projective variety of dimension n, and let F be a smooth projective variety parameterizing a family of proper subvarieties of dimension d on X. Let $E = \{(C, x) \in F \times X : x \in C\}$, and let $p: E \to F$ and $q: E \to X$ be the natural projections. The cohomological Abel–Jacobi mapping is the morphism of Hodge structures of type $(-d, -d)$ defined by the composition

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