Motions of Trivial Links, and Ribbon Knots

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Introduction

A motion of a link consists of an isotopy of the link through its ambient space that ultimately returns the link to itself. By reducing this notion to the classical dimension, a classical braid can be considered the trace of a motion of a point set in a plane.

The set of motions of a link naturally forms a group, called a *motion group*. It is not easy to describe the motion group explicitly for a given link; Goldsmith [G1; G2] calculated motion groups for a trivial link and torus links in the 3-sphere. One might conjecture that the motion group of a trivial link of n-spheres in S^{n+2} would have the same structure as that in the classical dimension. In this paper, we give a result (Theorem 2.2) on motions of a trivial link of two components in general dimensions that might support its motion group structure.

Using our result on motions of a trivial link, we can define an invariant of ribbon presentations of knots. A *ribbon presentation* is geometric information defining a knot to be a ribbon, which is introduced and studied in [M2], [M3], [NN], and [Ya]. Specifically in this paper, we treat 1-fusion ribbon presentations—that is, a description $\mathcal{R}$ of a knot as obtained from the trivial link of two n-spheres in S^{n+2} by connecting them with a pipe. Then the centerline of the pipe links two components of the trivial link, and we can naturally assign a word w in two letters by reading off the linking of the centerline and the trivial link. Associated with this word w, we define a certain equivalence class $W(\mathcal{R})$ in two leters, and show that this turns out to be an invariant of 1-fusion ribbon presentations (Theorem 4.1).

A ribbon knot possibly has distinct ribbon presentations. We construct a ribbon knot having arbitrarily many different ribbon presentations of 1-fusion in general dimension (Theorem 4.4), and we use our invariant $W(\mathcal{R})$ for distinguishing those ribbon presentations. The first example of a knot

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