Boundary Limits of the Bergman Kernel and Metric

HAROLD P. BOAS, EMIL J. STRAUBE, & JIYE YU

1. Introduction and Results

Bergman computed, for certain special domains, the precise boundary behavior of the Bergman kernel function $K(z)$ on the diagonal and of the Bergman metric (see [3; 4]). Subsequently, Hörmander [25] showed for any bounded strongly pseudoconvex domain in $\mathbb{C}^n$ that the limit of $K(z)d(z)^{n+1}$ at the boundary (where $d(z)$ is the Euclidean distance from z to the boundary) equals the determinant of the Levi form times $n!/4\pi^n$. Diederich [10; 11] computed the boundary limit of the Bergman metric for strongly pseudoconvex domains. Later, Klembeck [29], using Fefferman's asymptotic expansion for the Bergman kernel function [16], found the boundary limit of the holomorphic sectional curvature of the Bergman metric in strongly pseudoconvex domains. (See [19] and [27] for more on the curvature of the Bergman metric.)

Although there has been considerable progress in estimating the size of the Bergman kernel and metric on weakly pseudoconvex domains of finite type (see e.g. [6; 13; 14; 21; 23; 24; 32; 34; 35; 36]), boundary limits in the sense of Hörmander's result are not well understood. Examples of Herbort [20; 22] show that in general the growth of the Bergman kernel function is not an algebraic function of the distance to the boundary. In this paper we show that the kernel function, weighted by a suitable power of the distance to the boundary, does have a nontangential limit for a large class of weakly pseudoconvex domains of finite type in $\mathbb{C}^n$. We also show the existence of limits for the Bergman metric and its holomorphic sectional curvature. Moreover, we evaluate these limits in terms of the corresponding Bergman invariants of an unbounded local model of the finite type domain; in favorable cases, these admit explicit computation.

Our method is based on Bergman's original approach of minimum integrals. The idea, following [43; 28], is first to localize the minimum integrals as in [25]; then to blow up the domain via dilations, in the spirit of the scaling method [2; 39]; and finally to observe that the minimum integrals of the dilates approach the minimum integrals of the local model. In the last step,

Received September 6, 1994. Revision received March 14, 1995.
The first two authors were partially supported by NSF grant number DMS-9203514.
Michigan Math. J. 42 (1995).