Interpolating Varieties and Counting Functions in $\mathbf{C}^n$

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1. Introduction

In this paper, we shall study the extension problem of an analytic function satisfying growth conditions on an analytic variety of codimension 1 to an entire function on $\mathbf{C}^n$ satisfying the same kind of growth conditions, where

$$V = f^{-1}(0) := \{\zeta \in \mathbf{C}^n : f(\zeta) = 0\}$$

defined as the zero set of an entire function f in a given weighted space. If each analytic function on V with growth conditions has such a global extension, V is then called an *interpolating variety* (defined below).

The above interpolation problem has been studied by many authors (see e.g. [Be, De, Oh, Sk]) and is related to harmonic analysis (cf. [BT3]). An open problem [BT2] is to find geometric interpolation conditions that depend only on the geometry of varieties. When $n = 1$, it has been shown that V is an interpolating variety for A_p (resp. A_p^0) if and only if

$$N(|\zeta|, \zeta, V) \leq Ap(\zeta) + B, \quad \zeta \in V, \tag{1.1}$$

for some $A, B > 0$ (resp. $N(|\zeta|, \zeta, V) = o\{p(\zeta)\}, \zeta \in V, \zeta \to \infty$) (see [BL, BLV, Sq]), where $N(|\zeta|, \zeta, V)$ is the counting function of V and p is the given weight (defined below). The advantage of this condition is that one can determine whether V is an interpolating variety by estimating the geometric quantity $N(|\zeta|, \zeta, V)$, which depends only on the "value distribution" of V. Note that the counting function is one of the most important quantities in studying value distribution of holomorphic mappings in one and several complex variables (see e.g. [Gr]). It is a natural goal to consider whether it could give geometric interpolation conditions in the higher-dimensional case. The paper is concerned with this problem. We shall prove that (1.1) can still be used to give a sufficient interpolation condition. However, a counterexample shows that (1.1) is no longer necessary for interpolation in the higher-dimensional case.

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