Cyclicity and Approximation by Lacunary Power Series

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Introduction

Let $\ell^p = \ell^p(\mathbb{Z}_+)$, $1 \le p \le \infty$, be the classical Banach space of all p-summable complex sequences $x = \{x_0, x_1, x_2, \dots\}$. Define the operators S and T on ℓ^p by

$$S\{x_0, x_1, x_2, \dots\} = \{0, x_0, x_1, \dots\},$$
$$T\{x_0, x_1, x_2, \dots\} = \{x_1, x_2, x_3, \dots\}.$$

These operators are called, respectively, *the forward* (right) and *the backward* (left) *shifts.*

In dealing with S and T it is convenient to consider the spaces ℓ_A^p of those power series (or analytic functions in the unit disk $\mathbb{D}$) that are the discrete Fourier transforms of the elements of ℓ^p:

$$\ell_A^p = \left\{ f = \sum_{k=0}^{\infty} \hat{f}(k) z^k : \|f\|_p = \left(\sum_{k=0}^{\infty} |\hat{f}(k)|^p \right)^{1/p} < \infty \right\}, \quad 1 \le p < \infty;$$
$$\ell_A^\infty = \left\{ f = \sum_{k=0}^{\infty} \hat{f}(k) z^k : \|f\|_\infty = \sup_{k \ge 0} |\hat{f}(k)| < \infty \right\}$$

(we always consider ℓ_A^∞ to be endowed with the weak* topology). Clearly, the space ℓ_A^2 coincides with the well-known Hardy space H^2. We use the same letters S and T for the corresponding operators on ℓ_A^p:

$$Sf = zf,$$
$$Tf = \frac{f - f(0)}{z}.$$

A subspace E of ℓ_A^p is called *S-invariant* if $SE \subset E$, and *T-invariant* if $TE \subset E$ (subspace always means a closed linear subspace).

We denote by q the conjugate exponent of p (i.e., the number determined by $1/p + 1/q = 1$), and we identify the space ℓ_A^q with the dual space of ℓ_A^p (if $p \ne \infty$), with duality defined by

$$\langle f, \phi \rangle = \sum_{k=0}^{\infty} \hat{f}(k) \hat{\phi}(k), \quad f \in \ell_A^p, \ \phi \in \ell_A^q.$$

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