The Subspace Problem for Weighted Inductive Limits of Spaces of Holomorphic Functions

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The aim of the present article is to solve in the negative a well-known open problem raised by Bierstedt, Meise, and Summers in [BMS1] (see also [BM1]). We construct a countable inductive limit of weighted Banach spaces of holomorphic functions, which is not a topological subspace of the corresponding weighted inductive limit of spaces of continuous functions. As a consequence, the topology of the weighted inductive limit of spaces of holomorphic functions cannot be described by the weighted sup-seminorms given by the maximal system of weights associated with the sequence of weights defining the inductive limit. The main step of our construction shows that a certain sequence space is isomorphic to a complemented subspace of a weighted space of holomorphic functions. To do this we make use of a special sequence of outer holomorphic functions and of the existence of radial limits of holomorphic bounded functions in the disc.

Weighted spaces and weighted inductive limits of spaces of holomorphic functions on open subsets of $\mathbb{C}^N$ ($N \in \mathbb{N}$) arise in many fields, such as linear partial differential operators, convolution equations, complex and Fourier analysis, and distribution theory. Since the structure of general locally convex inductive limits is rather complicated and many pathologies can occur, the applications of weighted inductive limits have been restricted. The reason was that it did not seem possible to describe the inductive limit, its topology, and in particular a fundamental system of seminorms in a way that permits direct estimates and computations. In the theory of Ehrenpreis [Eh] of "analytically uniform spaces", the topology of certain weighted inductive limits of spaces of entire functions, which are the Fourier–Laplace transforms of spaces of test functions or ultradistributions, were required to have a fundamental system of weighted sup-seminorms. Berenstein and Dostal [BD] reformulated the problem in a more general setting and used the term "complex representation". This corresponds exactly with the term "projective description" used by Bierstedt, Meise, and Summers [BMS1], which is the one we will utilize in this paper. In [BMS1] it was proved that countable weighted inductive limits of Banach spaces of holomorphic functions on arbitrary open subsets G of $\mathbb{C}^N$ admit such a canonical projective description by weighted

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