# Cores of Ideals in 2-Dimensional Regular Local Rings

CRAIG HUNEKE & IRENA SWANSON

## 1. Introduction

The main result of this paper is the explicit determination of the core of integrally closed ideals in 2-dimensional regular local rings. The core of an ideal $I$ in a ring $R$ was introduced by Judith Sally in the late 1980s and was alluded to in Rees and Sally's paper [RS]. Recall that a reduction of $I$ is any ideal $J$ for which there exists an integer $n$ such that $JI^n = I^{n+1}$ [NR]. In other words, $J$ is a reduction of $I$ if and only if $I$ is integrally dependent on $J$. An ideal is integrally closed if it is not a reduction of any ideal properly containing it.

(1.1) DEFINITION. The core of an ideal $I$, denoted core($I$), is the intersection of all reductions of $I$.

In general, the core seems extremely difficult to determine and there are few computed examples. A priori, it is not clear whether it is zero. However, one can show that, in general, the core always contains a power of $I$. A proof of this for Buchsbaum rings can be found in [Tr, Prop. 5.1].

It is quite natural to study the core, partly due to the theorem of Briançon and Skoda (see [BS; LS; LT; L4; HH; RS; Sa; AH1; AH2; AHT]). A simple version of this theorem states that if $R$ is a $d$-dimensional regular ring and $I$ is any ideal of $R$, then the integral closure of $I^d$ is contained in $I$. In particular, the integral closure of $I^d$ is contained in core($I$). It is an important question to understand how the core of $I$ relates to $I$. More generally, we are interested in approximating general $m$-primary ideals in local rings $(R, m)$ by intersections of parameter ideals. We hope our results in dimension 2 will provide insight into the nature of the core in higher dimensions.

Some of the open questions regarding the core are as follows.

(a) If $I$ is integrally closed, is core($I$) also integrally closed?
(b) If the completion $\hat{R}$ of $R$ is equidimensional, does core($I$)$\hat{R}$ equal core($\hat{I}$)? More generally, how does the core behave under faithfully flat maps?

Received June 1, 1994. Revision received September 1, 1994.
The authors thank the NSF for partial support.
Michigan Math. J. 42 (1995).