A Harmonic Quadrature Formula Characterizing Bi-Infinite Cylinders

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1. Introduction and Results

In the following let $K_r = \{x \in \mathbb{R}^m : |x| < r\}$ be an open ball of radius $r > 0$ centered at the origin. $|\cdot|$ always denotes the Euclidean norm and λ_m the m-dimensional Lebesgue measure. Here m and (later on) n will be natural numbers.

We are concerned with *harmonic quadrature formulas*. The prototype is Gauss's well-known mean value formula:

For every harmonic and integrable function $h: K_r \to \mathbb{R}$, the following mean value property holds:

$$\int_{K_r} h \, d\lambda_m = \lambda_m(K_r) \cdot h(0).$$

For a $(1+n)$-dimensional strip $(-r, r) \times \mathbb{R}^n$, the following quadrature formula is true for harmonic and integrable functions $h: (-r, r) \times \mathbb{R}^n \to \mathbb{R}$:

$$\int_{(-r,r)\times\mathbb{R}^n} h \, d\lambda_{1+n} = \lambda_1(K_r) \cdot \int_{\mathbb{R}^n} h(0, \xi) \, d\lambda_n(\xi)$$

(see [2] or [7]).

Now consider $m \geq 2$. For an $(m+n)$-dimensional bi-infinite cylinder $K_r \times \mathbb{R}^n \subset \mathbb{R}^{m+n}$, we shall prove a similar quadrature formula in Section 2, as follows.

THEOREM 1. *Let $h: K_r \times \mathbb{R}^n \to \mathbb{R}$ be harmonic and integrable on $K_r \times \mathbb{R}^n$. Then*

$$\int_{K_r\times\mathbb{R}^n} h \, d\lambda_{m+n} = \lambda_m(K_r) \cdot \int_{\mathbb{R}^n} h(0, \xi) \, d\lambda_n(\xi).$$

Open balls and open strips can even be characterized by harmonic quadrature. Indeed, Kuran [11] gave a simple proof of the following result:

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