On the Fitting Ideals in Free Resolutions

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Introduction

Throughout this paper, all rings are commutative with identity. If R is a ring and if $\phi: F \to G$ is a map of finitely generated free R-modules, then we define $I_i(\phi)$ $(i \geq 0)$ to be the ideal of R generated by the $i \times i$ minors of a matrix representing ϕ and the rank of ϕ, denoted by rank ϕ, to be the largest number t such that $I_t(\phi) \neq 0$. The ideals $I_i(\phi)$ are called the *Fitting ideals* of ϕ.

Let $(R, \mathfrak{m}, K)$ be a d-dimensional complete Noetherian local ring containing a field with maximal ideal $\mathfrak{m}$ and residue class field $K = R/\mathfrak{m}$. The purpose of this paper is to study a conjecture of C. Huneke concerning the behavior of Fitting ideals in free resolutions of finitely generated modules over R. In the meantime, a question about the annihilator ideal of the functor $\mathrm{Ext}_R^{d+1}(-,-)$ is also considered. In order to present these questions, more definitions are needed.

Let R be as above. Then, by Cohen structure theorem,

$$R \cong K[|X_1, \dots, X_n|]/(f_1, \dots, f_t)$$

for some indeterminates $X_1, \dots, X_n$ and some power series

$$f_1, \dots, f_t \in K[|X_1, \dots, X_n|].$$

Therefore, from this representation, we may define the Jacobian ideal of R to be $I_h(\partial(f_1, \dots, f_t)/\partial(X_1, \dots, X_n))R$, that is, the ideal of R generated by the image of $h \times h$ minors of the Jacobian matrix $(\partial(f_1, \dots, f_t)/\partial(X_1, \dots, X_n))$, where $h = \text{height}(f_1, \dots, f_t)$. Furthermore, we denote by $I_s(R)$ the ideal defining the singular locus of R; that is, $I_s(R) = \bigcap_{P \notin \operatorname{Reg} R} P$. If M is a finitely generated R-module then M is said to have a *well-defined rank r* if, for any $P \in \operatorname{Ass}(R)$, M_P is free and $\mu_P(M) = r$. Finally, we denote by $(\mathbf{F}., \phi.)$ the following acyclic complex of finitely generated free R-modules:

$$\cdots F_d \xrightarrow{\phi_d} F_{d-1} \xrightarrow{\phi_{d-1}} \cdots \xrightarrow{\phi_2} F_1 \xrightarrow{\phi_1} F_0.$$

Let us state the questions as follows.

CONJECTURE 1. Let $(R, \mathfrak{m}, K)$ be a d-dimensional complete Noetherian local ring containing a field and let J be the Jacobian ideal of R. Let M be a

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