The PL Fibrators among Aspherical Geometric 3-Manifolds

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Results of [D6] indicate that manifolds in a surprisingly extensive collection act as PL fibrators. This paper adds evidence for a claim that most manifolds, in a psychological rather than a strictly mathematical sense, have this desirable attribute. Among closed, orientable 2-manifolds the exceptions are known to consist only of the 2-sphere and the torus. Among closed, orientable 3-manifolds, those having either hyperbolic, Sol, or $SL_2(R)$ geometric structure are all PL fibrators, as are those with infinite first homology which are connected sums of nonsimply connected manifolds, provided the fundamental groups are residually finite. A long advance in an effort to classify 3-manifolds with this feature, the main result here (Theorem 3.4) shows that an aspherical, virtually geometric 3-manifold is a PL fibrator if it is one in codimension 2. Similar in tone and next in importance, Theorem 2.10 shows that any closed manifold with $(k-1)$-connected compact universal cover is a codimension-k PL fibrator if it is one in codimension 2.

To explain what all this means, we begin by setting forth the notation and fundamental terminology to be employed throughout: M is a connected, orientable, PL $(n+k)$-manifold, B is a polyhedron, and $p: M \to B$ is a PL map such that each $p^{-1}b$ has the homotopy type of a closed, connected n-manifold. When N is a fixed orientable n-manifold, such a (PL) map $p: M \to B$ is said to be *N-like* if each $p^{-1}b$ collapses to an n-complex homotopy equivalent to N. (This PL tameness feature imposes significant homotopy-theoretic relationships, revealed in [D6, Lemma 2.4], between N and preimages of links in B.) One calls N a *codimension-k PL fibrator* if, for every orientable $(n+k)$-manifold M and N-like PL map $p: M \to B$, p is an approximate fibration. Finally, if N has this property for all $k > 0$, one simply calls N a *PL fibrator.*

Remarkably, at this stage of development only two types of manifolds are known not to be PL fibrators: those that already fail in codimension 2, and those that have a sphere as Cartesian factor. The codimension-2 situation, reviewed extensively in the introduction to [D5], is fairly well understood and is not treated here.

Earlier work showing certain manifolds to be PL fibrators typically require the fundamental groups to have no nontrivial, Abelian normal subgroups. That accounts for the richness of information available about connected

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