Geometry of Operator Spaces

Å. Lima, E. Oja, T. S. S. R. K. Rao,*
& D. Werner

1. Introduction

The results in this paper aim at giving information on the position of $K(X, Y)$, the space of compact linear operators between Banach spaces X and Y, in $L(X, Y)$, the space of bounded linear operators.

It is known that for a number of spaces, for example $X = l^p$ and $Y = l^q$ $(1 < p, q < \infty)$, $K(X, Y)$ is an M-ideal in $L(X, Y)$ (the definition of an M-ideal is given in Section 2); thus the position of $K(X, Y)$ in $L(X, Y)$ vaguely resembles the position of c_0 in l^∞ in this case. We show in Section 2 that, in several instances, a necessary condition for $K(X, Y)$ to be an M-ideal in $L(X, Y)$ is that $K(l^1, Y)$ be an M-ideal in $L(l^1, Y)$; and we go on to investigate those Banach spaces Y for which the latter holds. We prove that such a space is a nonreflexive (unless finite-dimensional) Asplund space; in fact, it is even an M-ideal in its bidual. For the proof of the latter assertion we offer a characterization of M-ideals X in X^{**} which yields in particular that this property is separably determined. Moreover, we prove for a separable space with the metric compact approximation property that $K(l^1, Y)$ is an M-ideal in $L(l^1, Y)$ if and only if $K(X, Y)$ is an M-ideal in $L(X, Y)$ for every Banach space X. This class of Banach spaces, called (M_∞)-spaces in [29], was introduced and investigated in [30].

Section 3 deals with the problem of unique Hahn–Banach extensions from $K(X, Y)$ to $L(X, Y)$. The results in this section are motivated by two recent results. For a certain class of Banach spaces X that includes the l^p $(1 < p < \infty)$ spaces, it is proved in [29] that for any Banach space Y, every continuous linear functional on $K(X, Y)$ has a unique norm-preserving extension to a linear functional on $L(X, Y)$. On the other hand, one of us [24] has recently shown that if x is a denting point of the unit ball X_1 of X and y^* is a w^*-denting point of Y_1^* then the functional $x \otimes y^*$ has unique norm-preserving extension from $R(X, Y)$, the space of finite rank operators, to $L(X, Y)$.

We study the properties of a Banach space X for which, for a compact Hausdorff space Ω, extreme points in the unit ball of $K(X, C(\Omega))^*$ have unique

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