L_2 Cohomology of Pseudoconvex Domains with Complete Kähler Metric

HAROLD DONNELLY

1. Introduction

Let Ω be a bounded pseudoconvex domain with smooth boundary in C^n. The Bergman metric of Ω is a complete Kähler metric. Every biholomorphic automorphism of Ω induces an isometry relative to the Bergman metric. We use $\mathcal{H}_2^{p,q}(\Omega)$ to denote the space of square integrable harmonic (p,q) forms, associated to the Bergman metric. The following result was proved several years ago [5].

THEOREM 1.1. *If Ω is strictly pseudoconvex, then*

$$\dim \mathcal{H}_2^{p,q}(\Omega) = \begin{cases} 0 & p+q \neq n, \\ \infty & p+q = n. \end{cases}$$

Ohsawa has developed this work by giving both alternative proofs and applications to extension problems in the analysis of several complex variables [11; 12; 13].

More recently, Gromov [7] studied the L_2 cohomology of complete Kähler manifolds. Suppose that M is a complete, simply connected, Kähler manifold. Assume that the Kähler form $\omega = d\eta$, where η is bounded in L^∞ norm. Under these hypotheses, Gromov proved this next theorem.

THEOREM 1.2. *If M covers a compact manifold, then*

$$\dim \mathcal{H}_2^{p,q}(M) = \begin{cases} 0 & p+q \neq n, \\ \infty & p+q = n. \end{cases}$$

Theorems 1.1 and 1.2 have analogous conclusions. However, of all strictly pseudoconvex domains in C^n endowed with Bergman metrics, only the ball covers a compact manifold. Thus, the hypotheses of both theorems are only satisfied in a single example.

The purpose of the present article is to show that, nevertheless, the techniques of Gromov can be employed to give a more transparent proof of Theorem 1.1. Gromov's ideas lead to a sufficient condition for $\dim \mathcal{H}_2^{p,q}(\Omega) = 0$,

Received January 20, 1993. Revision received January 14, 1994.
The author was partially supported by NSF Grant DMS-9200225.
Michigan Math. J. 41 (1994).