The Index of Transversally Elliptic Operators on Locally Homogeneous Spaces of Finite Volume

JEFFREY FOX[1] & PETER HASKELL[2]

Introduction

There is a well-developed index theory of elliptic operators on compact manifolds. On noncompact manifolds a variety of approaches to index theory of elliptic operators has yielded interesting information under various assumptions. On a class of noncompact, but finite-volume, locally homogeneous spaces, elliptic differential operators descended from invariant operators on the associated homogeneous spaces can be used to define Fredholm operators with interesting indices. In general, to define the Fredholm operator one must restrict the elliptic operator to the "discrete summand" of a spectral decomposition determined by the Lie group used to define the locally homogeneous space. There is a large literature on this subject. A concise discussion of the aspects relevant to our paper appears in [Mo].

There is also an index theory of operators elliptic in directions transverse to group actions or to foliations, especially on compact manifolds [At; NZ; Si; Ve; Co; CS; HS]. In [FH2] we studied operators T invariant under and elliptic in directions transverse to locally free actions of noncompact Lie groups G on compact manifolds. Such an operator is not generally Fredholm, but each irreducible G-representation occurs with finite multiplicity in the kernels of the operator and its adjoint. In [FH2] we showed how to use the indices of elliptic operators on compact manifolds to calculate, for some irreducible G-representations β, the difference: multiplicity of β in kernel(T) minus multiplicity of β in kernel(T^*).

In the present paper we extend the above results to certain noncompact locally homogeneous settings. We give in Section 1 the precise assumptions under which we work, as well as an indication of the variety and complexity of examples that occur. In this introduction we describe our setting less carefully as follows. Let G_1 and G_2 be noncompact, connected, semisimple Lie groups. Assume that H is a compact subgroup of G_2 and that Γ is a lattice

Received November 17, 1992.

[1] Work supported by the National Science Foundation under grant no. DMS-8903472.
[2] Work supported by the National Science Foundation under grant nos. DMS-8901436 and DMS-9204275.

Michigan Math. J. 41 (1994).