Weakly Outer Polynomials

R. CHENG & S. SEUBERT

1. Introduction

The notion of a weakly outer function has its origins in the prediction theory of 2-parameter stationary random fields. Evidently, those stationary fields with the so-called commutation properties provide a natural medium in which to seek out multiparameter extensions of many classical 1-parameter results [5; 8; 11]. In [1; 2] it is proved that such a field possesses the "weak commutation property" if and only if its spectral density is the squared modulus of a weakly outer function in $H^2(\mathbf{T}^2)$ (a slightly weaker result is obtained in [8]). The related function theory and further applications to prediction are treated in [3].

In the present work, another prediction-theoretic result is obtained. It states that for weakly commutative stationary fields, the past, conditioned on the future (in some sense), is finite-dimensional if and only if the associated weakly outer function is a certain type of rational function. This, in turn, points to the need to characterize the weakly outer polynomials. A complete characterization in terms of zero sets is found.

2. Notation and Preliminaries

Let $\mathbf{D}$ be the open unit disc, and $\mathbf{T}$ the unit circle, in the complex plane $\mathbf{C}$. Normalized Lebesgue measure on $\mathbf{T}$ is written $d\sigma$, and $d\sigma_r$ is the associated product measure on the torus $\mathbf{T}^r$. By the symbols $N_*(\mathbf{D}^r)$ and $H^p(\mathbf{D}^r)$ we mean the usual Nevanlinna and Hardy classes of analytic functions on the polydisc $\mathbf{D}^r$ (see [6; 9]). We shall identify a function on the polydisc with its radial limit function on the torus, whenever the latter exists; likewise, an integrable function on the torus will be identified with its harmonic extension into the polydisc.

The symbol $\hat{}$ indicates a Fourier coefficient. Thus, if $f \in L^1(\mathbf{T}^2)$ then

$$\hat{f}_{m,n} = \int f(e^{is}, e^{it}) e^{-ims-int} \, d\sigma_2(e^{is}, e^{it}).$$

Received
Michigan Math. J. 41 (1994).