Automorphisms of Negatively Curved Polygonal Amalgams

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1. Introduction

In [13] it was shown that the free product with amalgamation of two "nice" (e.g. finite) groups has a very rigidly controlled automorphism group. The argument is based on the action of the amalgamated product on its associated tree. It is natural to ask if similar results hold for groups acting on higher-dimensional objects.

Polygonal amalgams of groups are the 2-dimensional analogues of free products with amalgamation. By studying the actions of negatively curved polygonal amalgams on their corresponding 1-connected 2-complexes, we are able to describe the automorphism group of the amalgam group. In particular, we show that the automorphism group of a negatively curved polygonal amalgam of finite groups is virtually a negatively curved polygonal amalgam of finite groups. It follows from our study of automorphisms that the automorphism group of a Coxeter group acting on the hyperbolic plane with compact fundamental domain has outer automorphism group a finite dihedral group (possibly trivial), and the full automorphism group is itself a Coxeter group.

We extend this analysis of automorphisms of negatively curved polygonal amalgams to the more general case of injective endomorphisms, and show that negatively curved polygonal amalgams of finite groups are co-Hopfian. Because polygonal amalgams where the edge groups generate the group are 1-ended, this result supports Gromov's statement in [11], "Γ *is not isomorphic to any of its proper subgroups.* Probably, the same is true for every word hyperbolic group Γ connected at infinity." (The italics are Gromov's.) We believe Gromov's statement is still an open question for arbitrary 1-ended word hyperbolic groups.

Most of the definitions can be found in Sections 2, 3, and 7. Section 4 describes triangles of groups, and the more general notion of polygons of groups, in more detail. Sections 5 and 6 establish geometric facts about some piecewise hyperbolic 2-complexes, analogous to standard results in the geometry of the hyperbolic plane. These are then applied in the final section to

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