A Remark on Quasiconformal Mappings on Carnot Groups

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1. Introduction

In a letter of May 1991, A. Koranyi and M. Reimann informed me that a result of theirs on the theory of quasiconformal mappings on the Heisenberg groups contradicted inequality (20.17) in my monograph [2] which is asserted there without proof and used in the proof of Proposition 21.3. The latter proposition deals with the extension of a certain mapping φ between hyperbolic space over the division algebra $\mathbf{K}(\mathbf{K} := \mathbf{R}, \mathbf{C}, \mathbf{H} :=$ quaternions, or $\mathbf{O} :=$ octonions) to their boundaries at infinity. The boundary map φ_0 had previously been proved to be a quasiconformal mapping over $\mathbf{K}$. Proposition 21.3 asserts that φ_0 is absolutely continuous on almost all curves of a specified type.

The boundary minus one point is the free action orbit of any maximal unipotent subgroup of the isometry group of the hyperbolic space. In case $\mathbf{K} = \mathbf{C}$, the unipotent group is the Heisenberg group; for a general $\mathbf{K}$, it is a two-step unipotent Carnot group.

Proposition 21.3 is an essential step in proving strong rigidity for locally hyperbolic spaces over $\mathbf{K}$. This paper offers a correction of the proof of Proposition 21.3 via bypassing the faulty inequality (20.17). The method used can be generalized directly to simplify the definition of quasiconformal mappings on two-step Carnot groups. In [2] the notion of quasiconformal mapping on the boundary of hyperbolic space is defined in terms of the boundary "semimetric". Subsequently, Pansu (in [3]), and Koranyi and Reimann (in an earlier version of [1]) studied a similar notion of quasiconformal mapping with respect to a "Carnot–Carathéodory metric", which required an extra "doubling hypothesis". In Section 4 it is pointed out that, as a result of the method used here, the extra doubling hypothesis is superfluous. This method was subsequently adopted by Koranyi and Reimann in [1].

2. Setting the Stage

Occurring in the proof of Proposition 21.3 are two commuting fibrations $\pi^{\mathbf{K}}$ and $\pi_{\mathbf{R}}$ by Hopf fibers and by quarter great $\mathbf{R}$-circles respectively. The point

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