Self-Duality and 4-Manifolds with Nonnegative Curvature on Totally Isotropic 2-Planes

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1. Introduction

In [MM], Micallef and Moore proved a beautiful result which gives a topological classification of simply connected compact manifolds with positive curvature on totally isotropic 2-planes, namely that they are homeomorphic to the sphere. In this paper we want to consider the case of nonnegative curvature on totally isotropic 2-planes (see Definition 3.2) for 4-dimensional compact manifolds. Our first result is the following theorem.

THEOREM 1. *Let M be an irreducible, simply connected compact 4-manifold. If M has nonnegative curvature on totally isotropic 2-planes then M is either homeomorphic to the sphere S^4 or biholomorphic to the complex projective space* $\mathbf{CP}^2$.

Also in [MM, p. 222], the authors investigated some commonly used curvature conditions which imply the nonnegativity of the curvature on totally isotropic 2-planes. (For brevity we denote this by NNC.) For the case of dimension 4, some other conditions will give NNC. For instance, the results of Seaman in [S1] imply that compact, positively curved, real 4-dimensional Kähler manifolds have NNC. Conformally flat 4-manifolds with nonnegative scalar curvature have NNC.

In this paper we will investigate some conditions on a half-conformally flat manifold which will imply nonnegativity of the curvature on totally isotropic 2-planes. For example, although the nonnegativity of the scalar curvature is a necessary condition (Proposition 3.3), Theorem 1 and Theorem B in [Po] combined show that it cannot be sufficient even for positive scalar curvature. We will give in the next theorem a condition in terms of the sectional curvatures which will be a sufficient condition.

THEOREM 2. *Let M^4 be a half-conformally flat manifold with nonnegative scalar curvature. Then M has nonnegative curvature on totally isotropic 2-planes if and only if for any orthonormal basis $\{e_i, e_j, e_m, e_k\}$ of the*

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