The Infinite Nielsen Kernels of Some Bordered Riemann Surfaces

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To Lipman Bers

1. Introduction

Let X be a Riemann surface of finite type (p, n, m), with $m \geq 1$. That means X can be conformally embedded in a closed Riemann surface Y of genus p so that $Y \setminus X$ consists of $m \geq 1$ disjoint closed disks and $n \geq 0$ additional points, called the *punctures* of X. There is a unique bordered Riemann surface $\bar{X}$ whose interior is X and whose border B is the union of m disjoint simple loops C_j, called the *boundary loops* of X.

Suppose X has negative Euler characteristic:

$$e(X) = 2 - (2p + n + m) < 0. \tag{1.1}$$

Then each boundary loop C_j is freely homotopic in $\bar{X}$ to a unique simple closed Poincaré geodesic C_j' in X, and C_j' and C_k' are disjoint if $j \neq k$. (These geodesics are defined using the complete Poincaré metric of curvature -1 on X, which puts the boundary loops C_j at infinite distance.) The Nielsen kernel of X is the interior $N(X)$ of the bordered Riemann surface obtained from X by removing the m annuli bounded by the pairs of freely homotopic loops C_j and C_j', $1 \leq j \leq m$.

Viewed as a Riemann surface in its own right, $N(X)$ has the same finite type (p, n, m) as X. We can therefore iterate the construction above, forming the nested sequence of Riemann surfaces

$$N^{k+1}(X) = N(N^k(X)) \subset N^k(X) \subset N^1(X) = N(X) \subset X, \quad k \geq 1.$$

Bers [2] suggested an investigation of the set

$$N^\infty(X) = \bigcap_{k=1}^{\infty} N^k(X),$$

called the *infinite Nielsen kernel* of X. That is a hard problem. The first progress was made by Wason [11] and Halpern [5; 6]. They compared lengths and distances of certain closed geodesics on X and $N(X)$, and studied the effect of iteration. Their results suggest that $N^\infty(X)$ is a rather thin set. That

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