# Global Integrability of the Jacobian and Quasiconformal Maps

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## 1. Introduction

Here we present alternate proofs to certain results arrived at in Astala and Koskela's recent article, "Quasiconformal mappings and global integrability of the derivative" [AK]. In addition, we examine how these new ideas shed light on some of the questions raised therein on the geometry of Gehring domains.

Denote the Jacobian matrix of $f$ at $x$ by $F(x)$ and its determinant by $J(x,f)$. Define

$$|f'(x)| = \sup_{h \in \mathbf{R}^n, |h|=1} |F(x)h|. \tag{1.1}$$

Let $D$ and $D'$ be domains in $\mathbf{R}^n$, $n \geq 2$. A homeomorphism $f: D \to D'$ is said to be *K-quasiconformal* if $f \in W^1_{n,\mathrm{loc}}(D)$ and

$$|f'(x)|^n \leq KJ(x,f) \text{ a.e. in } D. \tag{1.2}$$

Local integrability results of the following type are well known for quasiconformal maps [Ge]. If $f: D \to D'$ is $K$-quasiconformal and $E$ is any compact set in $D$, then there exists an exponent $p = p(n,K) > 1$ such that

$$\int_E (J(x,f))^p \, dm \leq M < \infty. \tag{1.3}$$

Here $M$ depends on $E$ and $f$.

In order to understand corresponding global integrability results, we need the following definitions.

DEFINITION 1.4. The *quasihyperbolic distance* between $x$ and $y$ in $D$ is given by

$$k_D(x,y) = \inf_\gamma \int_\gamma \frac{1}{d(z,\partial D)} \, ds,$$

where $\gamma$ is any rectifiable curve in $D$ joining $x$ to $y$. Here $\partial D$ denotes the boundary of $D$ and $d(z,\partial D)$ stands for the distance from $z$ to the boundary of $D$.

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