# Operators Defined on Projective and Natural Tensor Products

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## Introduction

This paper studies the behaviour of operators defined on the projective and natural tensor product of an $l_p$ space and an arbitrary Banach space $X$. The main result is the following (Theorem 1): If $E$ and $F$ are Banach spaces, $E$ not containing $l_1$, and all operators from $E$ into $l_p$ and all operators from $E$ into $F$ are compact, then all operators from $E$ into $l_p \hat{\otimes}_\pi X$ are compact. Two applications of the techniques involved in the proof of this result are considered: a study of the tensor stability (with respect to the projective and natural tensor product with an $l_p$-space) of the scale of operator ideals formed by the $p$-converging operators for $1 \leq p \leq +\infty$, and a vector-valued version of Pitt's theorem.

## Background

Throughout the paper $p^*$ denotes the dual number of $p$. We base our approach to the properties of natural and projective tensor products on the use of the representations of those spaces as sequence spaces. A sequence $(x_n)$ in a Banach space $X$ is said to be *weakly p-summable* ($p \geq 1$) if there is a $C > 0$ such that, for each $(\xi_n)$ in $l_{p^*}$,

$$w_p(\{x_n\})_n = \sup_k \left\{ \left\| \sum_{n=1}^{k} \xi_n x_n \right\| : \|(\xi_n)\|_{l_{p^*}} \leq 1 \right\} < +\infty$$

(here, if $p = 1$ then $c_0$ plays the role of $l_\infty$); it is said to be *absolutely p-summable* when $p \geq 1$ if

$$s_p(\{x_n\}_n) = \left[ \sum_{n=1}^{+\infty} \|x_n\|^p \right]^{1/p} < +\infty$$

(if $p = +\infty$ then the $l_p$ norm must be replaced by the sup norm); it is said to be *strongly p-summable* for $p \geq 1$ if

$$\sigma_p(\{x_n\}_n) = \sup \left\{ \left| \sum_{n=1}^{+\infty} f_n(x_n) \right| : w_{p^*}(\{f_n\}) \leq 1, (f_n) \in X^* \right\} < +\infty.$$

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