Extensions of Complex Varieties across C^1 Manifolds

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Introduction

More than twenty years ago, Shiffman [Sh] proved that a closed subset E in an open set $\Omega \subset \mathbf{C}^n$ with zero $(2k-1)$-dimensional Hausdorff measure does not obstruct complex varieties, in the sense that if V is a k-dimensional complex variety in $\Omega \backslash E$ and if $\bar{V}$ is the closure of V in Ω, then $\bar{V} \cap \Omega$ is also a k-dimensional complex variety in Ω. Intuitively speaking, the set E is too small to obstruct the variety V, since the topological dimensional of E does not exceed $2k-2$ while the boundary of V has dimension less than or equal to $2k-1$. Later on, Alexander [A] and Becker [Be] considered the case when E is a real-analytic set with dimension as large as $2k-1$ but, in addition, V has certain symmetric properties. In this paper, we consider the following problem: If E does not obstruct the variety V topologically (for the precise definition see 1.5 below), is it necessary that $\bar{V} \cap \Omega$ also be a complex variety? We give an affirmative answer to this question when E is a $(2k-1)$-dimensional C^1 manifold. In a subsequent paper we will show that the same conclusion holds when E is either a rectifiable curve or a high-dimensional rectifiable set subject to certain requirements. For a general closed set E, the problem is far from being solved.

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1. Basic Definitions and Main Result

We start with some basic definitions and properties of analytic varieties.

Suppose that Ω is an open subset in $\mathbf{C}^n$. A subset V of Ω is a *complex analytic variety* if, for each point p in Ω, there exists a neighborhood U of p and a family of functions that are holomorphic in U such that $V \cap U$ is the set of common zeros for the functions in the family.

A complex variety V is called *irreducible* if it cannot be decomposed as $V = V_1 \cup V_2$, where V_i are two distinct proper subvarieties of V. A variety is irreducible if and only if the set of its regular points, $V_{\text{reg}} = V - V_{\text{sing}}$, is a connected set. The irreducible components of the variety V are the sets $\bar{W}_i$, where the sets $\{W_i\}$ are the connected components of V_{reg}.

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