# Generalized Feynman Integrals via Conditional Feynman Integrals

DONG MYUNG CHUNG, CHULL PARK,
& DAVID SKOUG

## 1. Introduction

In various Feynman integration theories, the integrand $F$ of the Feynman integral is a functional of the standard Wiener (i.e., Brownian) process. In this paper we first define a Feynman integral for functionals of more general stochastic processes. Then, for various classes of functionals, we express this generalized Feynman integral as an integral operator whose kernel involves the conditional Feynman integral.

In defining various analytic Feynman integrals of $F$, one usually starts, for $\lambda > 0$, with the Wiener integral

$$\int_{C_0[0,T]} F(\lambda^{-1/2}x+\xi)\psi(\lambda^{-1/2}x(T)+\xi)m(dx),$$

and then extends analytically in $\lambda$ to the right-half complex plane. In this paper, our starting point is the Wiener integral

$$\int_{C_0[0,T]} F(\lambda^{-1/2}Z(x,\cdot)+\xi)\psi(\lambda^{-1/2}Z(x,T)+\xi)m(dx), \tag{1.1}$$

where $Z$ is the Gaussian process

$$Z(x,t)=\int_0^t h(s)\,dx(s) \tag{1.2}$$

with $h$ in $L_2[0,T]$, and where $\int_0^T h(s)\,dx(s)$ denotes the Paley-Wiener-Zygmund (PWZ) stochastic integral.

A very important class of functionals in quantum mechanics are functionals on Wiener space $C_0[0,T]$ of the form

$$F(x)=\exp\left\{\int_0^T \theta(s,x(s))\,ds\right\} \quad \text{and} \quad G(x)=F(x)\psi(x(T)) \tag{1.3}$$

for appropriate functions $\theta$ and $\psi$. Functionals like these have appeared in many papers, including [1–4; 7; 8; 14; 15; 19], involving various Feynman integration theories. In particular, Cameron and Storvick [8] obtain a formula

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