Weighted Spaces of Holomorphic Functions on Balanced Domains

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Dedicated to Professor George Maltese on the occasion of his 60th birthday

Weighted spaces $HV_0(G)$ and $HV(G)$ of holomorphic functions as well as weighted inductive limits $\mathcal{V}_0H(G)$ and $\mathcal{V}H(G)$ of spaces of holomorphic functions arise naturally in various applications of functional analysis, for example, in partial differential equations and convolution equations, complex and Fourier analysis, distribution theory, and spectral theory. However, many (structural) properties of these spaces which would be very helpful in concrete analytical investigations are rather hard to prove in a general context. In the present article, attention is restricted to (increasing systems V and decreasing sequences $\mathcal{V}$ of) *radial* weights on *balanced* domains $G \subset \mathbf{C}^N$ ($N \geq 1$), which makes it possible to apply arguments involving the Taylor series of holomorphic functions about zero. In this way, we obtain some progress, which is even interesting in the case of Banach spaces of this type.

More specifically, in Section 1 we use (Fejér's result on) the contractive properties of the Cesàro means of the Taylor series of functions in the disk algebra to derive remarkable consequences for spaces of holomorphic functions on arbitrary balanced open sets $G \subset \mathbf{C}^N$. Our method leads to simple proofs that the spaces $HV_0(G)$ and $\mathcal{V}_0H(G)$ have the bounded approximation property whenever they contain the polynomials, and that then the polynomials are dense. Similar results are also true for the larger spaces $HV(G)$ and $\mathcal{V}H(G)$, but only under certain (quite natural) weaker topologies. The bidualities $((HV_0(G))_b')_b' = HV(G)$ and $((\mathcal{V}_0H(G))_b')_i' = \mathcal{V}H(G)$ (which were established in [5] under slightly stronger hypotheses) actually hold in the present generality. The results of Section 1 serve as a basis for the developments in the subsequent sections.

Section 2 is devoted to a problem which had already been raised in [6] and [7] (and is part of a more general problem that has interested various authors): Can one interchange the inductive limit $\mathcal{V}_0H(G) = \text{ind}_n\, H(v_n)_0(G)$ and the ϵ-(tensor) product with an arbitrary Banach space X, in particular if $\mathcal{V}$ satisfies the condition that for each $n \in \mathbf{N}$ there is $m > n$ such that v_m/v_n vanishes at infinity on G? This is closely related to the important question of whether one can obtain a *projective description* $\mathcal{V}_0H(G,E) = H\bar{V}_0(G,E)$

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