On the Dimension of Harmonic Measure of Cantor Repellers

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1. Introduction

Let J be a Cantor repeller for a conformal map f. We prove that the harmonic measure of J has dimension strictly less than the Hausdorff dimension of J for J lying on a line.

Harmonic measure plays an important part in 1-dimensional complex analysis. Recently, the structure of harmonic measure of rather general plane sets has become much more comprehensible due to works of Carleson [C1; C2], Makarov [Ma1], and Jones and Wolff [JW1]. The deep analogy between the behaviour of sums of (almost) independent random variables and the behaviour of the Green function of a domain plays a crucial part in this subject. We refer the reader to [Ma2] for background. This analogy becomes still more conspicuous if a domain for which the harmonic measure is investigated has a regular self-similar structure. As Carleson [C2] showed, the methods of the ergodic theory turn out to be relevant in this case. This approach was used also in [P], [Z], and [MV]. In this work we will also use extensively ideas from [C2]. In some sense we continue here the study that was undertaken in [C2] and [MV].

It is worth noting that harmonic measure in a dynamical context appeared for the first time in Brolin's paper [Br], where it was established that backward orbits of a polynomial f are equidistributed with respect to the harmonic measure w of the unbounded component of the Julia set $J(f)$. Later this result was interpreted as the coincidence of w and the unique measure of maximal entropy of f ("maximal measure") in [L] and [M]. The Julia set of the polynomial f can be very complicated, but in one particular case it is simply a Cantor-like set—namely, when the orbits of all critical points $c_1, \ldots, c_{d-1}$ go to infinity. For such f the Julia set $J(f)$ represents an example of a Cantor repeller. But now we would like to note that the result of Brolin together with Manning's formula [Man] gives the following estimate of the dimension of harmonic measure on the Cantor-like Julia set of a polynomial f of degree d

$$\dim w = \frac{\log d}{\int_{J(f)} \log|f'|\, dw} < 1. \tag{1.1}$$

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