Proper Holomorphic Self-Mappings of Hartogs Domains in C^2

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It has been an open problem whether a proper holomorphic self-mapping of a smooth bounded pseudoconvex domain in C^n is biholomorphic [B]. In [A], Alexander showed that each proper holomorphic self-mapping of the unit ball is an automorphism. This result has been generalized to several cases; for example, in the case of strictly pseudoconvex domains, it is due to Pincuk [Pi]. Bedford and Bell [BB] verified the result in the case of smooth real-analytic pseudoconvex domains. Recently, we have been able to prove that Alexander's theorem remains true in the case of smooth pseudoconvex Reinhardt domains whose Levi determinant vanishes to finite order and in the case of Reinhardt domains with real-analytic boundary (not necessarily pseudoconvex) [P1; P2]. We note that all previous results involve a kind of finite type condition for weakly pseudoconvex boundary points. In this note, we shall provide a class of domains in which many points of infinite type may occur and Alexander's theorem remains valid.

To state our result, we need some basic notation. Let Ω be a smooth bounded domain in C^2 and let r be a smooth defining function of Ω defined in C^2 such that $\Omega=\{r<0\}$ and $dr\neq 0$ on $b\Omega$. Define a function $\Lambda_r\colon C^2\to R$ by

$$\Lambda_r=-\det\begin{pmatrix} 0 & r_{\bar z} & r_{\bar w} \\ r_z & r_{z\bar z} & r_{\bar z w} \\ r_w & r_{z\bar w} & r_{w\bar w}\end{pmatrix},$$

which we call the *Levi determinant*. Note that if Ω is pseudoconvex then $\Lambda_r(z)\geq 0$ for $z\in b\Omega$ and that the set $W(b\Omega)=\{z\in b\Omega;\ \Lambda(z)=0\}$ is precisely that of all weakly pseudoconvex boundary points of Ω. We will study pseudoconvex Hartogs domains in C^2 which admit a defining function of the form

$$r(z,w)=|w|^2+\phi(z),$$

where $\phi(z)$ is a real-valued smooth function on C, so that

$$\Omega=\{(z,w)\in C^2;\ |w|^2+\phi(z)<0\}.$$

Let $E=\{(z,0)\}\cap\Omega$. E is called the *base domain* of the Hartogs domain Ω. We see that $E=\{(z,0)\in C^2;\ \phi(z)<0\}$.

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