Two Metric Invariants for Riemannian Manifolds

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1. Examples and Main Theorems

In this short note we shall define two metric invariants for a closed metric manifold (i.e., a topological manifold with a metric). Each of them has its own topological significance. The purpose of this note is to associate each invariant with some familiar geometric invariants of Riemannian manifolds, for example, curvatures, diameter and volume. Our first metric invariant concerns the orbits of finite groups acting on metric manifolds. In [N], Newman proved that at least one orbit is not too small if the action of a finite group on a closed connected metric manifold is nice. More precisely, Newman proved the following theorem.

THEOREM (Newman). *If M is a closed topological manifold with a metric d, then there exists a positive number $\eta = \eta(M, d)$ depending only on M and d such that every finite group G acting effectively on M has at least one orbit of diameter at least η.*

For instance, it can be shown that the unit n-sphere S^n with its canonical metric d has $\eta(S^n, d) \geq 1$, and that the flat torus $(T^n, d) = (\mathbf{R}^n, \text{can})/\mathbf{Z}^n$ has $\eta(T^n, d) \geq \frac{1}{4}$.

Cernavskii [C] generalized Newman's theorem to the setting of finite-to-one open mappings on metric manifolds. We shall say that an open finite-to-one proper surjective map f (which is not a homeomorphism) from a metric manifold M to a metric space Y is a *pseudo-submersion,* and that $f^{-1}(f(x))$ is an orbit of f at x and is denoted by $O_f(x)$. In [MR], McAuley and Robinson expanded upon Cernavskii's result and obtained the following theorem.

THEOREM. *If M is a closed topological manifold with a metric d, then there is a positive number $\eta = \eta(M, d)$ such that if Y is a metric space and $f: M \to Y$ is a pseudo-submersion then there is at least one point $y \in Y$ with* $\operatorname{diam} f^{-1}(y) \geq \eta$.

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