Geodesic Excursions into Cusps in Finite-Volume Hyperbolic Manifolds

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0. Introduction

Throughout, $\mathfrak{M}^{d+1}$ will be a fixed, complete, noncompact Riemannian manifold of constant negative sectional curvature and finite volume. Given a point p on $\mathfrak{M}$, we denote by $S(p)$ the unit ball of the tangent space of $\mathfrak{M}$ at p, and for every $v \in S(p)$ let $\gamma_v(t)$ be the geodesic emanating from p in the direction v. In this paper, we study the long time behaviour of $\gamma_v(t)$.

Sullivan proved in [S] that for almost every direction $v \in S(p)$, one has

$$\limsup_{t\to\infty} \frac{\operatorname{dist}(\gamma_v(t), p)}{\log t} = \frac{1}{d},$$

where dist is the distance in $\mathfrak{M}$. On the other hand, for just a countable number of directions $v \in S(p)$,

$$\limsup_{t\to\infty} \frac{\operatorname{dist}(\gamma_v(t), p)}{t} = 1.$$

We give a result interpolating between these two.

Theorem 1. *For* $0 \le \alpha \le 1$,

$$\operatorname{Dim}\left\{v : \limsup_{t\to\infty} \frac{\operatorname{dist}(\gamma_v(t), p)}{t} \ge \alpha\right\} = d(1-\alpha).$$

Here and hereafter, Dim denotes Hausdorff dimension. Dimension refers here to the induced distance in $S(p)$. Also, we will use the notation M_α for α-dimensional content. We refer to [C] or [R] for definitions and background on these metrical notions.

Let $\mathbf{H}^{d+1}$ be the upper half plane of $\mathbf{R}^{d+1}$,

$$\mathbf{H}^{d+1} = \{(x_1, \dots, x_{d+1}) \in \mathbf{R}^{d+1} : x_{d+1} > 0\},$$

and let λ be the hyperbolic metric in $\mathbf{H}^{d+1}$,

$$d\lambda = \frac{|dx|}{x_{d+1}}.$$

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