Equivariant Harmonic Maps between Compact Riemannian Manifolds of Cohomogeneity 1

HAJIME URAKAWA

Dedicated to Professor Tadashi Nagano on his sixtieth birthday

Introduction

The aim of this paper is to develop the earlier works of Smith [12; 13] (see also [5; 7]) on symmetric harmonic maps between spheres and also his reduction theorem on equivariant harmonic maps. As applications, we construct explicitly (1) new harmonic maps from 2-flat tori into spheres, complex projective spaces, quaternion projective spaces or complex quadrics, (2) new harmonic maps from complex projective spaces into spheres, and (3) new nonholomorphic harmonic maps from complex projective spaces into other complex projective spaces.

At the present time, the only known method to construct harmonic maps from a higher-dimensional Riemannian manifold to another compact Riemannian manifold of positive curvature involves investigation of minimal submanifolds, holomorphic maps, or equivariant maps whose symmetry of large isometry group action reduces the Euler–Lagrange equation to an ordinary differential equation. We are concerned with the last one, and assume that both domain and target Riemannian manifolds are of cohomogeneity 1; that is, both admit the isometry group actions having orbits of codimension 1. Then we derive the ordinary differential equations of the equivariant harmonic maps between them (cf. §2), and solving these equations, we construct the above harmonic maps (cf. §§3–6).

ACKNOWLEDGMENT. This work was done during my stay at MSRI, Berkeley. I would like to express my thanks for its financial support and hospitality. I would like to thank Professors W. Y. Hsiang and Y. L. Xin, who gave me helpful suggestions.

1. Preliminaries

In this section, we prepare the materials which will be needed in the following arguments.

Received October 9, 1990.
Research at MSRI supported in part by NSF Grant DMS-8505550.
Michigan Math. J. 40 (1993).