Analytic Disks in Fibers over the Unit Ball of a Banach Space

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1. Introduction

Fix an infinite-dimensional complex Banach space $\mathfrak{X}$, with open unit ball B. We are interested in studying the uniform algebra $H^\infty(B)$ of bounded analytic functions on B, and its spectrum $\mathfrak{M} = \mathfrak{M}(B)$ consisting of the non-zero complex-valued homomorphisms of $H^\infty(B)$. The spectrum $\mathfrak{M}$ is fibered in a natural way over the closed unit ball $\bar{B}^{**}$ of the bidual $\mathfrak{X}^{**}$ of $\mathfrak{X}$. The projection of $\mathfrak{M}$ onto $\bar{B}^{**}$ is obtained by simply restricting $\varphi \in \mathfrak{M}$ to $\mathfrak{X}^*$, regarded as a subspace of $H^\infty(B)$. As a straightforward application of the Josefson–Nissenzweig theorem [Jo; Ni], it is shown in [ACG] that each fiber consists of more than one point and is in fact quite large. Our aim here is to prove a sharpened form of the Josefson–Nissenzweig theorem, and to use this to embed analytic disks in the fiber over 0. This stands in contrast to the situation in finite-dimensional Banach spaces, where one expects (and can prove under certain hypotheses) that the natural projection is one-to-one over the open unit ball $B = B^{**}$ and implements a homeomorphism of B and an open subset of $\mathfrak{M}$.

The Josefson–Nissenzweig theorem asserts that in any infinite-dimensional dual Banach space $\mathfrak{Z}$, there is a sequence $\{z_j\}$ converging weak-star to 0 such that $\|z_j\| = 1$. The accessory condition we require is that the distance from z_j to the linear span of the preceding z_i's tends to 1 as $j \to \infty$. Actually we prove that each z_j can be chosen to have unit distance from the linear span of the remaining z_i's, and in fact the z_j's can be taken as part of a unit biorthogonal system defined in Section 2.

In Sections 2 and 3 we establish the existence of unit biorthogonal systems having accessory properties involving weak or weak-star convergence to 0. In Section 4 we make some observations regarding infinite products in a uniform algebra. The embedding of analytic disks is accomplished in Section 5. In Section 6 we give conditions under which the unit ball of certain