A Simple Proof of a Theorem of Jean Bourgain

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In this note, we give a very simple proof (compared to earlier known proofs) of Bourgain's version of Grothendieck's theorem for the disk algebra: Every operator on the disk algebra with values in L_1 or L_2 is 2-absolutely summing and hence extends to an operator defined on the whole of C. As far as we know, the currently known proofs are essentially the original one in [B1], the simpler one in [BD], and several new proofs given recently by Kisliakov in [K1; K2]. This implies Bourgain's result that L_1/H^1 is of cotype 2. We also prove more generally that L_r/H^r is of cotype 2 for $0<r<1$.

We first recall the definition of a q-absolutely summing (in short, q-summing) operator for $1 \le q < \infty$. Let $u: X \to Y$ be an operator between two Banach spaces. We say that u is *q-summing* if there is a constant C such that, for all finite sequences $x_1, x_2, \dots, x_n$ in X, we have

$$(\sum \|u(x_i)\|^q)^{1/q} \le C \sup\{(\sum |x^*(x_i)|^q)^{1/q} \mid x^* \in X^*, \ \|x^*\| \le 1\}.$$

We denote by $\pi_q(u)$ the smallest possible constant C. Let us denote by A the disc algebra. Then, if $u: A \to Y$ is q-summing, by Pietsch's factorisation theorem there is a probability measure λ on the unit circle such that

$$\forall f \in A, \quad \|u(f)\| \le \pi_q(u) \left(\int |f|^q \, d\lambda \right)^{1/q}.$$

We refer for example to [P1] for more information on this notion.

We will prove the following theorem due to Bourgain.

BOURGAIN'S THEOREM. *There is a constant K such that any bounded operator $u: A \to l_2$ is 2-summing and satisfies*

$$\pi_2(u) \le K \|u\|.$$

Also, u extends to a bounded operator $\hat{u}: C(\mathbf{T}) \to l_2$ such that

$$\|\hat{u}\| \le K \|u\|.$$

Moreover, the same result holds for all operators $u: A \to Y$ if $Y = l_1$ or, more generally, whenever Y is a Banach space of cotype 2.

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