Plurisubharmonic Extremal Functions and Complex Foliations for the Complement of Convex Sets in $\mathbf{R}^n$

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In this paper we prove some properties of Siciak's extremal function Φ_E in the case of compact subsets of $\mathbf{R}^n$. In particular, we establish an interesting inequality for extremal functions of convex sets and present some corollaries that follow from this result. Moreover, we obtain effective formulas for the extremal function in a few interesting cases of convex symmetric sets and in the case of special nonsymmetric convex polyhedra. Finally, we present an effective continuous complex foliation of the domain $\mathbf{C}^n \setminus E$ (in the cases when we have explicit representation of the extremal function) by the leaves on which the plurisubharmonic extremal function u_E is harmonic.

1. Introduction and Statement of the Main Results

Let E be a compact set in $\mathbf{C}^n$. By $\Phi_E(z)$ $(\Phi(z,E))$ we denote Siciak's extremal function defined as follows:

$$(1.1) \qquad \Phi_E(z) = \sup\{|p(z)|^{1/\deg p} : p \in \mathbf{C}[w],\ \deg p \geq 1,\ \|p\|_E \leq 1\}$$

for $z \in \mathbf{C}^n$, where $\|p\|_E$ denotes the Čebyshev uniform norm $\|p\|_E = \sup|p|(E)$. For definition and applications of the extremal function we refer to Siciak's papers ([12], [13], [14]) and especially to Pawłucki and Pleśniak's papers ([9], [10]). The basic property of the extremal function just defined is contained in the following Zakharyuta–Siciak theorem (see [15] and [13]).

1.2. THEOREM. *If E is a compact subset of $\mathbf{C}^n$ then*

$$\Phi_E(z) = \exp u_E(z) \quad \text{for } z \in \mathbf{C}^n,$$

where $u_E(z) = \sup\{u(z) : u \in \mathcal{L}_n,\ u|_E \leq 0\}$ and $\mathcal{L}_n$ is the Lelong class of plurisubharmonic functions in $\mathbf{C}^n$ (briefly, PSH$(\mathbf{C}^n)$*) with logarithmic growth: $u(z) \leq \text{const} + \log(1+|z|)$, $z \in \mathbf{C}^n$.*

In this paper we consider the case when E is a compact set in $\mathbf{R}^n$. (Here we treat $\mathbf{R}^n$ as the subset of $\mathbf{C}^n$ such that $\mathbf{C}^n = \mathbf{R}^n + i\mathbf{R}^n$). Let us denote by g the Joukowski transformation: $g(z) = \frac{1}{2}(z + \frac{1}{z})$ for $z \in \mathbf{C} \setminus \{0\}$. Let $h : \mathbf{C} \setminus [-1,1] \to \mathbf{C}$

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