A Nonpermutational Integral Relation Algebra

H. ANDRÉKA,* I. DÜNTSCH, & I. NÉMETI*

1. Introduction

Tarski and Jónsson raised the question whether every integral representable relation algebra (RA) was representable over a group. McKenzie [MK1; MK2] answered this question in the negative. In fact, McKenzie introduced the notion of a permutational relation algebra, and he showed that every group representable RA is permutational, but that not every permutational RA is group representable. Moreover, he showed that the class $\mathcal{G}$ of all group representable RAs is not finitely axiomatizable relative to the class $\mathcal{P}$ of all permutational RAs. He then raised the problem of whether a nonpermutational representable integral relation algebra exists. In this paper, we answer this question in the affirmative. We also prove that the class of all permutational relation algebras is not finitely axiomatizable over the class of representable integral relation algebras.

1.1. NOTATION AND DEFINITIONS. A relation algebra

$$\mathfrak{A} = (A, +, \cdot, -, ;, ^{-1}, 0, 1, 1')$$

is a structure of type $(2, 2, 1, 2, 1, 0, 0, 0)$, where

R1 $(A, +, \cdot, -, 0, 1)$ is a Boolean algebra;

R2 $(A, ;, ^{-1}, 1')$ is an involuted monoid; and

R3 for all $a, b, c \in A$, the conditions

$$(a ; b) \cdot c = 0, \quad (a^{-1} ; c) \cdot b = 0, \quad (c ; b^{-1}) \cdot a = 0$$

are equivalent.

For history and context of the theory of relation algebras, the reader is invited to consult [TG] or [J].

For a nonempty set U, we set $V = U \times U$ and consider the following operations on $\mathfrak{P}(V)$, the power set of V:

(1) the Boolean operations $\cup$, $\cap$, $-$, together with the constants $\emptyset$ and $U \times U$.

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