The Null Blow-Up of a Surface in Minkowski 3-Space and Intersection in the Spacelike Grassman

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Introduction

A smoothly immersed compact orientable surface $Q$ in 3-dimensional Minkowski space can be decomposed into a disjoint union $D^+ \cup D^0 \cup D^-$, where the induced metric is positive definite on $D^+$, degenerate on $D^0$, and indefinite on $D^-$. Along $D^0$, the line orthogonal to $Q$ is also tangent to $Q$. Imposing natural transversality conditions on this configuration stratifies the surface so $SP \subset D^0 \subset Q$, where the set $SP$ of stall points is contained in the set $D^0$ of stall curves that is embedded in $Q$. The stratification is defined as the loci, where the orthogonal line bundle is tangent to the next lower stratum. In [K2] we constructed a Gauss map for $Q$ into the 2-sphere, $g: Q \to S^2$, with degree $\pm \frac{1}{2} \chi(Q)$. Here we construct a Gauss map for $D^0$ into the compactified spacelike Grassman, $cg: D^0 \to S^1 \times S^1$. In this context points of $SP$ correspond to intersection points of $cg$ with the diagonal in $S^1 \times S^1$. In this paper we establish a formula relating: the degree of $g$, $\chi(D^+)$, and the intersection number of $cg$ with the diagonal (Theorem 4). As a consequence we have two integral inequalities that can be used to characterize simple configurations (Theorem 6). We then construct the null blow-up NB of $Q$. This is a compact folded double cover of $D^- \cup D^0$, $\rho: NB \to D^- \cup D^0$, with an oriented line field $L$ whose zero points are exactly $SP \subset D^0$. This null blow-up is completely determined by the first fundamental form on $Q$, and can be thought of as a completion space for null geodesics in $Q$ (i.e., a blow-up space for the singularities in the null geodesic ODE). We then show that the sum of the indices at these zero points is the intersection number of $cg$ with the diagonal in $S^1 \times S^1$ (Corollary 8). Since this line field $\rho$-projects to null subspaces in $D^-$, this corollary links purely extrinsic properties (a Gauss map for $D^0$) with purely intrinsic properties (the global dynamics of null pre-geodesics in $D^- \cup D^0$). Furthermore, since these zero points can be viewed as a degenerate type of "conjugate point", we have a new link between conjugacy in the null geodesic ODE and global properties of the underlying manifold.

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