On the Steenrod Homology Theory of Compact Spaces

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1. Introduction

For compact metric spaces, Steenrod [31] defined homology groups based on "regular cycles." On the category $\mathbf{A}_{CM}$ of compact metric pairs the Steenrod homology theory H_* has many good properties. It satisfies the seven Eilenberg–Steenrod axioms; for a wide variety of coefficient groups it is isomorphic to the Čech homology theory; it is exact under every exact coefficient sequence; and it is proved in [25] that H_* has the following modified form of the continuity property. Let

$$(X_1, A_1) \leftarrow (X_2, A_2) \leftarrow \cdots$$

be an inverse sequence of compact metric pairs with the inverse limit (X, A). Then there is an exact sequence

$$0 \to \varprojlim_i{}^{(1)} H_{n+1}(X_i, A_i, G) \to H_n(X, A, G) \to \varprojlim_i H_n(X_i, A_i, G) \to 0$$

for every integer n and coefficient group G, where $\varprojlim^{(1)}$ is the first derived functor of $\varprojlim$.

The Steenrod homology has important applications in geometric topology and operator theory [9; 14]. The final argument in favour of the Steenrod homology is the well-known Steenrod duality theorem:

If A is an arbitrary closed subset of the sphere S^{n+1}, then for $0 < q < n$ the Steenrod homology group $H_q(A, G)$ is isomorphic to the Čech cohomology group $\check{H}^{n-q}(S^{n+1} \backslash A, G)$.

Note that Sitnikov [27] defined the Steenrod homology groups for metric spaces and extended the Steenrod duality theorem to arbitrary subsets of the sphere S^{n+1}.

The axiomatic characterization of the Steenrod homology theory on the category $\mathbf{A}_{CM}$ of compact metric pairs was obtained by Milnor [25]. Milnor characterized the Steenrod homology theory with the seven Eilenberg–Steenrod axioms together with the invariance axiom under a relative homeomorphism and the cluster axiom. Skljarenko [28] also obtained a characterization

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