Rigidity Theorems for Foliations by Surfaces and Spin Manifolds

JAMES L. HEITSCH[1] & CONNOR LAZAROV[2]

1. Introduction

This paper is devoted to applications of the Lefschetz fixed point theorem for foliations [HL1; HL2]. The applications presented here are generalizations of two classical theorems: the finiteness of the automorphism group of a compact oriented surface of genus greater than 1, and the rigidity of compact spin manifolds with nonvanishing $\hat{A}$ genus [AH].

A foliation of a compact connected oriented Riemannian manifold is rigid if no compact connected Lie group acts nontrivially as a group of isometries taking each leaf to itself. If the foliation admits an invariant transverse measure then we can define the foliation Euler number and $\hat{A}$ genus. Proposition 3.1 says that a foliation by oriented surfaces with negative foliation Euler number is rigid. Proposition 3.2 says that a foliation by spin manifolds with nonzero foliation $\hat{A}$ genus is rigid if we insist that the Lie group of isometries preserves the leafwise spin structure. The proofs involve the application of our foliation Lefschetz theorem to the leafwise de Rham and Dirac complexes.

We also give examples of foliations which satisfy the hypotheses of Propositions 3.1 and 3.2 but whose leaf-preserving isometry group (resp., leaf-preserving isometry group preserving the leafwise spin structure) is still not finite.

In Section 2 we review the Lefschetz theorem for foliations. In Section 3 we state and prove our main theorems, and in Section 4 we present our examples. Section 5 contains the proof of Proposition 2.2 and some remarks about the leafwise de Rham and signature complex. We would like to thank John Wood for a number of helpful conversations.

2. Review of the Lefschetz Theorem

We recall some of the material from [HL1]. Let M be a compact connected oriented Riemannian manifold of dimension m and F a codimension-q oriented foliation.

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