Minimal Hypersurfaces Foliated by Spheres

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1. Introduction

Let M^n be an n-dimensional submanifold of $\mathbf{R}^{n+1}$. If there is a 1-parameter family of hyperplanes of $\mathbf{R}^{n+1}$ whose intersections with M^n are round spheres, then we refer to M^n as being foliated by spheres. If M^n is an (open) subset of $N^n \subset \mathbf{R}^{n+1}$, and the intersections with a family of planes are pieces of round spheres, then we say that M^n is foliated by pieces of spheres.

This article consists of two main results and a corollary. First, if $M^n \subset \mathbf{R}^{n+1}$ ($n \geq 3$) is a minimal submanifold and M^n is foliated by spheres in parallel hyperplanes, then M^n is rotationally symmetric about an axis containing the centers of all the spheres. Second, if $M^n \subseteq \mathbf{R}^{n+1}$ ($n \geq 3$) is a minimal submanifold and M^n is foliated by pieces of spheres, then the hyperplanes containing these spheres must be parallel. M^n is not assumed complete. However, from our two main results, if such an M^n is complete then it is a higher-dimensional catenoid. The author would like to thank Richard Schoen for helpful discussions.

In an 1867 article by Riemann and Hattendorff [12], it was shown that a minimal surface in $\mathbf{R}^3$ that is foliated by circles in parallel planes must be either a piece of a catenoid or the example now called the "Riemann staircase." In 1869, Enneper [4] showed that if a minimal surface is foliated by circles or by circular arcs, then the planes containing the circles or circular arcs must be parallel. A lengthier discussion of these results is available in the new English edition of Nitsche's book [9].

In 1956, Shiffman considered the related problem with boundaries [14]. If a minimal surface $M^2 \subseteq \mathbf{R}^3$ is bounded by convex curves in parallel planes, and if M^2 is topologically an annulus, then the intersections of M^2 with all other parallel planes are also convex curves. Similarly, if the boundaries are circles in parallel planes then the intermediate cross-sections must be circles.

There is a conjecture of William Meeks that the topological assumption is unnecessary in Shiffman's results (cf. [6, p. 87]).

In 1983, Schoen [13] developed a version of Alexandrov reflection [1] that applies to minimal submanifolds with boundary. In our first result we will

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