Holomorphic Extension of Proper Meromorphic Mappings

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Introduction

Forstnerič [F1] has proved that every proper holomorphic mapping from a ball in C^n to a ball in C^N $(N \geq n \geq 2)$ that is sufficiently smooth on the closure of the ball is in fact a rational mapping. He left open the possibility that such a mapping could be indeterminate at a point of the boundary sphere. Cima and Suffridge have shown recently that this does not occur, and hence every such mapping extends to be holomorphic in a neighborhood of the closed ball. They prove the following local result.

THEOREM [CS]. *Suppose that B is the open unit ball in C^n and that U is a neighborhood of a point $p \in bB$. Suppose that $F: U \to C^N$ is a meromorphic mapping whose restriction to B maps B holomorphically into the open unit ball in C^N $(N \geq n)$, and that for each point $q \in U \cap bB$, $\|F(z)\| \to 1$ as $z \to q$ $(z \in B)$. Then F extends to a holomorphic mapping in some neighborhood of p. As a consequence, F is rational.*

In this note we prove the following more general result.

THEOREM 1. *Suppose that M is a real analytic (nonsingular) real hypersurface in C^n, U is a neighborhood of a point $p \in M$, and Ω is the portion of U lying on one side of M. Suppose that $F: U \to C^N$ is a meromorphic mapping whose restriction to Ω maps Ω holomorphically into the open unit ball in C^N $(N \geq n)$, and that for each point $q \in U \cap M$, $\|F(z)\| \to 1$ as $z \to q$ $(z \in \Omega)$. Then F extends to a holomorphic mapping in some neighborhood of p.*

Note that we make no geometric assumptions about the hypersurface other than its real analyticity. Here is the idea of the proof. Since the ring of germs of holomorphic functions at a point $p \in C^n$ is a unique factorization domain, we may assume that F is of the form f/g where no factor of g divides all the components of f. Certainly F would extend holomorphically past M at p if $g(p)$ did not vanish. We prove that if $g(p) = 0$, then some factor of g would necessarily divide each component of f. First we prove that it is sufficient to

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