Interpolating Blaschke Products and Factorization in Douglas Algebras

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Introduction

The following problem of Guillory, Izuchi, and Sarason is proven: Let B be a Douglas algebra and let u be a unimodular function in B which does not vanish identically on any nontrivial Gleason part in B. If q is a function in B whose zero set contains that of u, then u divides q^N for some $N \in \mathbf{N}$. By using function-theoretic methods we shall also generalize a recent theorem of Tolokonnikov on zero sets of ideals in H^∞.

Let H^∞ be the Banach algebra of all bounded analytic functions in the open unit disk $\mathbf{D} = \{z \in \mathbf{C}: |z| < 1\}$ and let $M(H^\infty)$ denote its maximal ideal space. For $m, x \in M(H^\infty)$, let $\rho(m,x) = \sup\{|f(x)|: f(m) = 0, \|f\| = 1\}$ denote the pseudohyperbolic distance of the points m and x in $M(H^\infty)$. By Schwarz–Pick's lemma, $\rho(z,w) = |(z-w)/(1-\bar{z}w)|$ if $z, w \in \mathbf{D}$. Let

$$P(m) = \{x \in M(H^\infty): \rho(m,x) < 1\}$$

be the Gleason part of $m \in M(H^\infty)$. Defining m to be equivalent to x, $m \sim x$, if $\rho(m,x) < 1$ then one can show [4, p. 402] that $\sim$ is an equivalence relation in $M(H^\infty)$. Thus the Gleason parts of two points are either disjoint or equal.

A Gleason part P is called an *analytic disk* if there exists a continuous, bijective map L of $\mathbf{D}$ onto P such that $\hat{f} \circ L$ is analytic in $\mathbf{D}$ for every $f \in H^\infty$, where $\hat{f}$ denotes the Gelfand transform of $f \in H^\infty$.

In his famous paper [8], Hoffman showed that any Gleason part $P(m)$ in $M(H^\infty)$ is either a single point or an analytic disk. Moreover, the latter occurs if and only if $m \in \mathbf{D}$ or lies in the (weak-*-) closure of an interpolating sequence in $\mathbf{D}$, that is, in the closure of a sequence $\{z_n\}$ satisfying

$$\inf_{m \in \mathbf{N}} \prod_{\substack{n \in \mathbf{N} \\ n \neq m}} \rho(z_n, z_m) \geq \delta > 0.$$

This leads us to the following definition.

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