Duality of Bloch Spaces and Norm Convergence of Taylor Series

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1. Introduction

Let X be a Banach space of analytic functions on the open unit disk $\mathbf{D}$ in the complex plane. We always assume that the polynomials are dense in X. Given f in X, let

$$f(z)=\sum_{k=0}^{\infty} a_k z^k$$

be the Taylor expansion of f. For any integer $n \geq 1$, let

$$f_n(z)=\sum_{k=0}^{n} a_k z^k$$

be the nth Taylor polynomial of f. It is natural to ask the following question: When does $\{f_n\}$ converge to f in the norm topology of X? We will consider the question for the following spaces in this paper: H^p spaces and VMOA; weighted Bergman spaces; Besov spaces; and the little Bloch space. We give the definitions of these spaces first.

For $1 \leq p < +\infty$, the Hardy space H^p consists of analytic functions f on the open unit disk $\mathbf{D}$ such that

$$\sup_{0<r<1} \frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p \, d\theta < +\infty.$$

It is well known that each function in H^p has boundary values almost everywhere on the unit circle. We will not distinguish between functions in H^p and their boundary values. Note that the norm of a function in H^p is precisely the (normalized) Lebesgue L^p norm of its boundary value function on the circle. VMOA is the predual of H^1 under the complex integral pairing

$$\langle f, g\rangle = \frac{1}{2\pi} \int_0^{2\pi} f(\theta)\overline{g(\theta)} \, d\theta.$$

Again we will not distinguish between functions in VMOA (of the disk) and their boundary values. A function on the unit circle is in VMOA if and only if it is the Szegö projection of a continuous function (see VI.5 of [3] or Theorem 8.4.7 of [6]). For $1 \leq p < +\infty$ and $\alpha > -1$, the weighted Bergman space

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