Polynomial Proper Holomorphic Mappings between Balls, II

JOHN P. D'ANGELO

Introduction

The purpose of this paper is to give a complete classification of all polynomial proper holomorphic maps between balls in complex vector spaces. The results here are extensions of the author's results in [D2; D4], and clarify the partial results in many other papers [CS2; Fa1; Fa2; Fo3; R3; W]. Furthermore, generalizations of these results apply in the rational case. The rational case is the natural one, because of the result of Forstneric [Fo2] that a sufficiently differentiable proper holomorphic map between balls is necessarily rational. The author has obtained partial results that he believes will lead to a complete classification of the rational proper maps. These will appear elsewhere. It is probably impossible to give such a classification of the proper maps that are not smooth at the boundary. See [Dor; H] for the construction of such proper maps.

The first result of the present paper gives a factorization of a proper polynomial map between balls. The author gave another version of this in [D2], but the present statement and proof are more transparent. The proof is the same without regard to the range or domain dimension, including the 1-dimensional case, or even the degree of the polynomial. The operations involved are either linear transformations, tensor products, or inverses of these. Analysis of the proof gives a complete description. The tensor product operation is as follows. Suppose that f is a holomorphic map from C^n to C^N, and that A is a linear subspace of C^N of dimension k. Let z denote the identity operation on C^n. We form a new holomorphic map

$$E_{(A,z)}f\colon C^n \to C^{N+k(n-1)}$$

as follows. Write $f = f_A \oplus f_{A^\perp}$ for the orthogonal direct sum decomposition of f determined by that on its range C^N, and define the tensor product operation by

$$E_{(A,z)}f = (f_A \otimes z) \oplus f_{A^\perp}.$$

See Section I for more details. Let B_n denote the unit ball in C^n. If we omit the dependence on A in the notation, our first result can be stated as follows.

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