An Algorithm for 2-Generator Fuchsian Groups

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Introduction

The purpose of this paper is to present a geometrically based algorithm for deciding whether or not two elements of PSL(2, **R**) generate a non-elementary discrete group. There is, however, an obvious difficulty with the word "algorithm," for that suggests a procedure that can, at least in principle, be programmed to run on a computer. The difficulty has to do with elliptic elements; there is no effective way to decide that an elliptic element does not have finite order. If we regard an algorithm as a procedure, involving computations in some field, that necessarily ends after finitely many steps, then we do indeed have such an algorithm, provided our field of computations includes all standard computations involving real numbers, including arithmetic operations, computation of the inverse cosine, and computations involving logarithms.

However, if we take the point of view that an algorithm is something that can, at least in principle, be programmed to run on a computer, then we can say that we have an algorithm to decide if two matrices in GL(2, **Z**), with positive determinant, generate a non-elementary free discrete subgroup of PSL(2, **R**).

The problem of finding criteria for discreteness of Fuchsian groups has been the source of considerable activity; our list of references includes only those that are specific to 2-generator groups (and not all of them), as opposed to more general criteria. Jørgensen's inequality [J] yields a necessary condition; sufficient conditions in some cases were given by Lyndon and Ullman [LU]; and necessary and sufficient conditions for the case of two parabolic generators were given by Beardon [B1]. Necessary and sufficient conditions, in the form of an algorithm, were given by Purzitsky ([P1], [P2], [P3]), Rosenberger ([R1], [R2], [R3], [R4], [R5]), Purzitsky and Rosenberger [PR], and by Kern-Isberner and Rosenberger [KR]. Their approach is primarily

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