Semicocycles and Weighted Composition Semigroups on H^p

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1. Introduction

We consider semigroups $(T_t)_{t\geq 0}$ on the Hardy space H^p of the unit disc $\mathbf{D}$, which are of the form

(1.1) $T_t: H^p \to H^p, \quad T_t f(z) = h_t(z) f(\Phi_t(z)) \quad (t \geq 0, f \in H^p, z \in \mathbf{D})$

with suitable analytic functions $\Phi_t: \mathbf{D} \to \mathbf{D}$ and $h_t: \mathbf{D} \to \mathbf{C}$. We suppose that $(\Phi_t)_{t\geq 0}$ is a semiflow (sometimes called semigroup) of analytic functions; that is, the mapping $t \mapsto \Phi_t(z)$ is continuous for every $z \in \mathbf{D}$, $\Phi_0(z) \equiv z$ and $\Phi_{t+s}(z) = \Phi_t(\Phi_s(z))$ for all $z \in \mathbf{D}$ and $t, s \in [0, \infty)$. An application of Vitali's theorem shows the joint continuity of the mapping $(z, t) \mapsto \Phi_t(z)$. We often write Φ instead of $(\Phi_t)_{t\geq 0}$. Semiflows are studied very comprehensively by Berkson and Porta [1].

In this paper we discuss the manner in which properties of semigroups $(T_t)_{t\geq 0}$ of the form (1.1) are related to the properties of the functions h_t.

DEFINITION 1. Let Φ be a semiflow. A family $(h_t)_{t\geq 0}$ of analytic functions $h_t: \mathbf{D} \to \mathbf{C}$ is called a *semicocycle for* Φ if

(i) the mapping $t \mapsto h_t(z)$ is continuous for every $z \in \mathbf{D}$,
(ii) $h_{t+s} = h_t \cdot (h_s \circ \Phi_t)$ for $t, s \geq 0$, and
(iii) $h_0 \equiv 1$.

$(h_t)_{t\geq 0}$ is said to be

continuous, if the mapping $(t, z) \mapsto h_t(z)$ is continuous,
differentiable, if for every $z \in \mathbf{D}$ the mapping $t \mapsto h_t(z)$ is differentiable, and
bounded, if every h_t is bounded $(t \geq 0)$.

By using Vitali's theorem one can show that a bounded semicocycle is continuous. If Φ is a semiflow and $(h_t)_{t\geq 0}$ a bounded semicocycle for Φ, then the family $(T_t)_{t\geq 0}$, given by (1.1), is a semigroup of bounded linear operators on H^p.

Let $\omega: \mathbf{D} \to \mathbf{C}$ be an analytic function satisfying $\omega \not\equiv 0$. If all zeros of ω are in the set $\{z \in \mathbf{D}: \Phi_t(z) = z$ for all $t \in [0, \infty)\}$ of fixed points of Φ, then

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