An Approximation Theorem for Szegö Kernels and Applications

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1. Introduction

We begin by explaining the title of this paper. Consider the following question: Let K be a distributional kernel on a compact CR manifold, for example, a compact hypersurface in $\mathbf{C}^n$. We ask what kind of conditions on K are sufficient to guarantee that K differs from the Szegö kernel only by a smooth function? Our answer provides an approximation theorem for Szegö kernels. This question of approximation arises naturally from the study of the Szegö kernel, since finding the Szegö kernel explicitly is almost impossible for most CR manifolds. Before making a precise statement, we introduce two applications of the main theorem which constitute motivations for this work.

APPLICATION 1. The first application is to the question of a localization of the Szegö kernels. Let Ω_1 and Ω_2 be two bounded pseudoconvex domains in $\mathbf{C}^n$ with smooth boundaries $b\Omega_j$. Suppose that $\Omega_2 \subset \Omega_1$ and $b\Omega_1 \cap b\Omega_2 \neq \emptyset$. If S_j is the Szegö kernel for Ω_j $(j=1,2)$, is $S_1 - S_2$ smooth in the interior of $(b\Omega_1 \cap b\Omega_2) \times (b\Omega_1 \cap b\Omega_2)$? The answer is yes, under a certain type of condition (see Corollary 4.3 for a precise statement). An analogous question on the Bergman kernels was resolved by Fefferman by an elegant trick [4].

APPLICATION 2. The second application is related to the study of the Szegö kernel for domains in $\mathbf{C}^3$. Let Ω be a bounded pseudoconvex domain with a smooth boundary. Suppose that a portion of Ω is defined by the defining function

$$\rho(z) = p_1(z_1, \bar{z}_1) + p_2(z_2, \bar{z}_2) - \operatorname{Im} z_3,$$

where p_j is a subharmonic but not harmonic polynomial. Let

$$\Omega_1 = \Omega \cap \{(0, z_2, z_3)\} \quad \text{and} \quad \Omega_2 = \Omega \cap \{(z_1, 0, z_3)\}.$$

We want to exploit the relationship between the Szegö kernels for Ω_1, Ω_2, and Ω. It turns out that the Szegö kernel for Ω differs by a smooth function from a kind of convolution of the Szegö kernels for Ω_1 and Ω_2 (see Theorem 5.1).

Received September 18, 1989.
Michigan Math. J. 37 (1990).