Isomorphisms of Alg $\mathcal{L}_n$ and Alg $\mathcal{L}_\infty$

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Let $\mathcal{H}$ be a complex Hilbert space and let $\mathcal{L}_{2n}$ ($\mathcal{L}_{2n+1}$) be the subspace lattice of orthogonal projections generated by $\{[e_1], [e_3], \dots, [e_{2n-1}], [e_1, e_2, e_3], [e_3, e_4, e_5], \dots, [e_{2n-3}, e_{2n-2}, e_{2n-1}], [e_{2n-1}, e_{2n}]\}$ (respectively, $\{[e_1], [e_{2i+1}], [e_{2i-1}, e_{2i}, e_{2i+1}]: i = 1, 2, \dots, n\}$) with an orthonormal basis $\{e_1, e_2, \dots, e_{2n}\}$ ($\{e_1, e_2, \dots, e_{2n+1}\}$).

In this paper the following are proved:

(1) If Φ: Alg $\mathcal{L}_{2n} \to$ Alg $\mathcal{L}_{2n}$ is an isomorphism, then there exists an invertible operator T in Alg $\mathcal{L}_{2n}$ such that $\Phi(A) = TAT^{-1}$ for all A in Alg $\mathcal{L}_{2n}$.

(2) If Φ: Alg $\mathcal{L}_{2n+1} \to$ Alg $\mathcal{L}_{2n+1}$ is an isomorphism, then there exists an invertible operator S in Alg $\mathcal{L}_{2n+1}$ such that either $\Phi(A) = SAS^{-1}$ or $\Phi(A) = SUAUS^{-1}$, where U is a $(2n+1) \times (2n+1)$ matrix whose $(k, 2n-k+2)$-component is 1 for $k = 1, 2, \dots, 2n+1$ and all other entries are 0.

(3) A map Φ: Alg $\mathcal{L}_\infty \to$ Alg $\mathcal{L}_\infty$ is an isomorphism if and only if there exists an invertible operator (not necessarily bounded) T such that $\Phi(A) = TAT^{-1}$ for all A in Alg $\mathcal{L}_\infty$.

1. Introduction

The study of non-self-adjoint operator algebras on Hilbert space was begun in 1974 by Arveson [1]. Recently, such algebras have been found to be of use in physics, in electrical engineering, and in general systems theory. Of particular interest to mathematicians are reflexive algebras with commutative lattices of invariant subspaces. The algebras Alg $\mathcal{L}_n$ and Alg $\mathcal{L}_\infty$ are important classes of such algebras. These algebras possess many surprising properties related to isometries, isomorphisms, cohomology, and extreme points. In this paper, we shall investigate the isomorphisms of these algebras.

First, we introduce the terminologies used in this paper. Let $\mathcal{H}$ be a complex Hilbert space and let $\mathcal{A}$ be a subset of $\mathcal{B}(\mathcal{H})$, the class of all bounded operators acting on $\mathcal{H}$. If $\mathcal{A}$ is a vector space over $\mathbf{C}$ and if $\mathcal{A}$ is closed under the composition of maps, then $\mathcal{A}$ is called an algebra. $\mathcal{A}$ is called a self-

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